

**A DELAYED TB-COVID-19
CO-INFECTION MODEL WITH
OPTIMAL CONTROL ANALYSIS**

EDITH KATHURE KIUNYE

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ABSTRACT

Tuberculosis (TB) remains to be a threat affecting millions of people worldwide annually. This burden has been worsened by the emergence of COVID-19, and the coexistence of TB and COVID-19 has put more pressure on already strained health services. Although the public health efforts to manage the transmission of COVID-19 were essential, they resulted in socioeconomic effects, including decreased productivity, economic turmoil and unemployment. Epidemiologically, regions with high TB prevalence overlap creating conditions that increase the risk of co-infection and intensify public health vulnerability. For this reason, mathematical modeling of COVID-19 has shifted from merely creating epidemic curves only to examining more complex transmission dynamics and disease interaction. This work extends a deterministic TB–COVID-19 co-infection model into a delayed formulation by incorporating time lags related to both the diagnosis of active TB and the start of treatment. This approach captures a more realistic representation of healthcare system constraints and patient progression processes. This study seeks to identify cost-effective mitigation strategies for curbing the transmission of TB–COVID-19 co-infection through optimal intervention costs. Qualitative analysis is used to ascertain the model's validity, including the positivity of solutions and the well-posedness of the model. The extended optimum control concern is formed and the analysis done using Pontryagin's Maximum Principle, leading to the characterization of optimal intervention policies that minimize disease burden while reducing the implementation costs. Numerical simulations shows that combining the three measures i.e. vaccination, physical distancing and early treatment reduces the prevalence compared to using one method. The results also shows that time delays in TB diagnosis and treatment increases the disease transmissions.