

FORECASTING KENYA'S PUBLIC DEBT USING TIME SERIES ANALYSIS

OBWOGE FRANKLINE KERARO

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DECLARATION

This research project is my original work and has not been presented elsewhere for a degree or any other award.

Signature Date.....

Obwoye Frankline Keraro,
Department of Mathematics and Statistics.
B531/1348/2019

This research project has been submitted for examination with our approval as the University Supervisors.

Signature Date

Dr. Zakayo N. Morris,
Department of Mathematics and Statistics
University of Embu.

Signature Date.....

Dr. Dominic M. Kitavi,
Department of Mathematics and Statistics
University of Embu.

DEDICATION

I dedicate this research to my parents, my dearest father Bishop Benjamin Gisemba Keraro and my dearest mother Patriciah Nyanchoka Keraro whose unwavering love, encouragement and sacrifice have been the foundation of all my achievements. I will not forget the difficult times we went through as a family when my dearest mother was involved in an accident in October 2020 that led to the amputation of her leg. Despite of her pain and long stay in hospital to March 2021, she encouraged me to continue with this work. This motivation never left my mind. My Dad remained strong with the family and always prayed for better days, He supported us both emotionally and financially and united us.

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LIST OF ABBREVIATION AND ACRONYMS

ACF	Autocorrelation Function
AIC	Akaike Information Criteria
ARIMA	Autoregressive Integrated Moving Average
ARMA	Autoregressive Moving Average
BIC	Bayesian Information Criteria
GDP	Gross Domestic Product
HES	Holt Exponential Smoothing
IMF	International Monetary Fund
LSE	Least Square Estimator
MA	Moving Average
MAE	Mean Absolute Error
MAPE	Mean Absolute Percentage Error
MLE	Maximum Likelihood Estimator
PACF	Partial Autocorrelation Function
RMSE	Root Mean Square Error

LIST OF SYMBOLS

φ	phi
ϕ	phi variant
σ	sigma
β	beta
ξ	xi
κ	kappa
γ	gamma
ε	epsilon
ρ	rho
θ	theta
ψ	psi
α	alpha
υ	upsilon

DEFINITION OF KEY TERMS

COVID-19	This infectious disease affects the respiratory system that originated from Wuhan China in December 2019.
Decade	This is a period of ten years
Forecasting	Is the estimation of future events based on the availability of past and present information.
GDP	Gross Domestic Product is the measure of the monetary value of final goods and services
MLE	Maximum Likelihood Estimate is the method of parameter estimation in model fitting
Public debt	This is the amount of money that a government owes to outside debtors.
R software	This is a statistical programming language used in data analysis
Secondary data	This is the data that has been collected by someone other than the primary user.
Shunning Away	This means keeping away or avoiding something

ABSTRACT

Public debt management and forecasting remain challenging for developing economies, including Kenya, where accurate predictions are essential for sustainable fiscal planning. This study aimed to analyze and forecast Kenya's public debt using two time series forecasting approaches: the Autoregressive Integrated Moving Average model and the Holt exponential smoothing model. The study sought to evaluate the performance of these models to determine the most efficient forecasting method for Kenya's debt forecasting. The research employed a cross-sectional study design, utilizing public debt data from the Central Bank of Kenya spanning January 2010 to December 2023. The methodology involved initial data preprocessing, stationarity testing, and pattern analysis, followed by dividing the data into training and testing sets. Both models were fitted to the training data, with parameters optimized through minimization of the Akaike Information Criterion and smoothing parameters. Results revealed that the Autoregressive Integrated Moving Average model demonstrated superior performance in forecasting domestic debt, with a Root Mean Square Error of 0.02649721 compared to 0.0311399 for the Holt exponential smoothing model. For external debt forecasting, the Holt exponential smoothing model showed marginally better results. In forecasting total public debt, the Autoregressive Integrated Moving Average model again proved more accurate, with a Root Mean Square Error of 0.05710133 compared to 0.06144849 for the Holt model. Based on these findings, the study recommends using the Autoregressive Integrated Moving Average model for forecasting domestic and total public debt in Kenya, while the Holt exponential smoothing method for external debt forecasting. Regular reassessment of model performance is encouraged to maintain accuracy as debt patterns evolve. Future research should consider incorporating multiple economic variables, exploring advanced time series models, and integrating debt sustainability frameworks to enhance forecasting accuracy.

CHAPTER ONE

INTRODUCTION

1.1 Background Information

Public debt has significantly influenced economies worldwide, presenting both opportunities and challenges for national development. While it enables increased government spending and investment, it also introduces substantial financial challenges (Alloza et al., 2020; Andrés et al., 2017; Burriel et al., 2020; Leal et al., 2007). According to Barro (1979), public debt emerges when government revenue from taxation and other sources fails to meet expenditure requirements, leading to a gradual accumulation of debt obligations. This fiscal imbalance occurs when the average revenue stream cannot sustain the level of government spending, necessitating borrowing to bridge the gap between income and expenditure.

Kenya's economy has faced considerable challenges in recent years, resulting in declining living standards for its citizens (Makau et al., 2018). The economic instability has discouraged both domestic and foreign investment, further compounding the country's financial difficulties. Multiple studies have identified key factors contributing to Kenya's growing public debt, including persistent inflation, public sector inefficiencies, currency devaluation, and increasing unemployment rates (Canova & Ciccarelli, 2009; Dinh, 2020; Makau et al., 2018; Ngugi, 2016; Wanjiru, 2020; William et al., 2019).

The scale of Kenya's public debt has expanded dramatically over the past decade. Central Bank of Kenya data indicates an increase from 1.3 trillion Kenyan shillings in November 2010 to 7.3 trillion in November 2020, representing an approximate 82% rise (Makau et al., 2018). The government's pursuit of development resources has necessitated both domestic and external borrowing. The COVID-19 pandemic further escalated this trend, with Kenya securing an additional 250 billion Kenyan shillings from the International Monetary Fund by March 2021. By June 2021, the total public debt reached 7.7 trillion Kenyan shillings, marking a twelve-fold increase from 0.6 trillion Kenyan shillings two decades prior.

Recent studies examining debt management in East African Community (EAC) member states have highlighted various aspects of regional financial dynamics. Murungi et al. (2023) investigated the relationship between government debt, interest rates, fiscal policy, and economic growth within the EAC, emphasizing the importance of sound fiscal and debt management for regional development. Within the Kenyan context, Njenga (2024) applied time series analysis to examine domestic credit growth patterns, providing valuable insights into the configuration of domestic credit structures.

The implications of external public debt on private investment in Kenya have been extensively studied by Machagua and Naikumi (2023), who emphasized the need for foreign borrowing to be accompanied by conditions that foster private investment growth. Additional research by Erick et al. (2023) has highlighted the critical relationship between proper debt management, fund allocation, and economic growth outcomes. The broader context of external public debt in developing Sub-Saharan African countries was examined by Otieno (2023), covering the period from 1980 to 2018, which provided crucial insights into the effects of high external indebtedness and the importance of controlling external credit facilities.

Significant contributions to understanding Kenya's debt dynamics have emerged from recent research. Engefu et al. (2024) conducted regression analysis examining the relationship between public debt and various economic indicators, underscoring the importance of appropriate policy interventions. Maugu et al. (2023) investigated the correlation between public debt levels and interest rates across selected EAC member countries, illuminating the consequences of high public debt on investment, borrowing, and economic stability. Complementary research by Kijjambu et al. (2023) examined debt sustainability sources in Uganda using public debt dynamics modeling, providing valuable insights for regional debt management strategies.

The current study addresses Kenya's public debt challenges through comparative analysis of two prominent forecasting models: the Autoregressive Integrated Moving Average (ARIMA) and Holt Exponential Smoothing models. The ARIMA methodology, developed by Box and Jenkins (Montgomery et al., 2008), requires stationary data series for accurate forecasting. A series achieves stationarity when its autocovariance remains

time-independent with a constant mean (Mutwiri, 2019). The Holt exponential smoothing method, created by Peter Winters, specifically addresses time series with outliers, minimizing the impact of random fluctuations (Box et al., 1970; Habibur et al., 2016; Munarsih & Saluza, 2020). This dual-model approach enables comprehensive analysis of domestic, external, and total public debt patterns in Kenya.

The significance of this research lies in its potential to inform policy decisions through accurate debt forecasting. By analyzing historical debt patterns and projecting future trends, this study aims to provide policymakers and stakeholders with robust analytical tools for debt management. The comparative analysis of ARIMA and Holt Exponential Smoothing models offers insights into their relative effectiveness in forecasting different components of Kenya's public debt, supporting evidence-based decision-making in fiscal policy development.

The increasing complexity of Kenya's public debt situation, coupled with global economic uncertainties, underscores the importance of accurate forecasting methodologies. This research contributes to the existing body of knowledge on public debt management while providing practical tools for fiscal policy implementation. Through systematic analysis of debt patterns and careful evaluation of forecasting models, this study seeks to enhance understanding of Kenya's public debt dynamics and support sustainable economic growth strategies.

1.2. Statement of the problem

Kenya has experienced an unprecedented surge in both domestic and external public debt over recent decades (Makau et al., 2018). While this borrowing has facilitated public investment and development projects, the long-term implications for economic stability and sustainable growth remain concerning. The rapid accumulation of public debt presents a critical challenge for fiscal policy management and economic planning in Kenya.

Previous studies have employed various analytical approaches to understand Kenya's public debt dynamics. Regression analysis has been predominantly used to examine the relationship between public debt and economic indicators (Engifu et al., 2024), while panel data analysis has explored the broader implications of debt accumulation across East African economies (Murungi et al., 2023). However, these methodological approaches

have significant limitations. Regression analysis, while useful for understanding relationships between variables, fails to capture the temporal dynamics and future trajectories of debt accumulation. Panel data analysis, though comprehensive in regional comparison, often overlooks country-specific debt patterns and unique economic circumstances.

The existing literature reveals conflicting evidence regarding the impact of public debt on economic growth. Some studies suggest positive effects through increased public investment and infrastructure development (Alloza et al., 2020; Andrés et al., 2017), while others document negative consequences of high debt levels on economic performance (Erick et al., 2023; Otieno, 2023). This contradictory evidence complicates policy formulation and debt management strategies.

The critical challenge lies in the absence of reliable forecasting mechanisms for Kenya's public debt trends. Current approaches have proven inadequate in providing accurate projections, leaving policymakers without the necessary tools for effective debt management planning. This limitation is particularly problematic given the distinct characteristics of domestic and external debt components, each requiring specific analytical consideration (Machagua & Naikumi, 2023).

Time series analysis offers distinct advantages over traditional analytical methods in debt forecasting. Unlike regression or panel data analysis, time series models can capture temporal patterns, seasonal variations, and dynamic relationships within debt data. These models account for the sequential nature of debt accumulation and can identify underlying patterns those other methods might overlook. The Autoregressive Integrated Moving Average (ARIMA) model, for instance, has demonstrated effectiveness in analyzing domestic credit growth patterns in Kenya (Njenga, 2024), while the Holt Exponential Smoothing (HES) model excels in handling data with outliers and irregular patterns.

The lack of comprehensive time series analysis in Kenya's public debt forecasting has created a significant knowledge gap. While studies have employed various economic models to understand debt dynamics (Maugu et al., 2023), these approaches often fail to provide accurate short-term and medium-term debt projections. This limitation hampers

the ability of policymakers to develop proactive debt management strategies and implement timely interventions.

Furthermore, the absence of comparative analysis between different forecasting methodologies has left uncertainty about which approaches are most suitable for Kenya's specific context. This research gap is particularly critical given the complex nature of public debt, which comprises both domestic and external components, each potentially requiring different forecasting approaches for optimal accuracy.

This study addresses these limitations by employing both ARIMA and HES time series models for public debt forecasting. These models offer sophisticated analytical capabilities that can capture the dynamic nature of debt accumulation while providing more reliable projections than traditional methods. By comparing these two approaches, the study aims to identify the most effective forecasting methodology for different components of Kenya's public debt.

The findings contributed to both theoretical understanding and practical application in public debt management. For policymakers and stakeholders, accurate forecasting models are essential tools for developing evidence-based policies that can help control debt levels while promoting sustainable economic growth. This research thus addresses a critical need in Kenya's economic planning and fiscal policy framework, providing the analytical foundation for more effective debt management strategies.

1.3 Justification of the Study

The study of Kenya's public debt forecasting is justified by several critical gaps in current research and methodological approaches. Previous studies have primarily focused on analyzing historical debt patterns using regression analysis and panel data methods, leaving a significant gap in forecasting methodologies. While these traditional approaches provide insights into past debt relationships, they fail to offer reliable predictions of future debt trajectories, which are essential for proactive policy formulation.

The application of time series analysis in Kenya's public debt forecasting represents an underexplored area of research. Despite the demonstrated success of time series models in financial forecasting across various economic contexts, their application to Kenya's

public debt management has been limited. This methodological gap necessitates a systematic evaluation of time series approaches, particularly the ARIMA and Holt Exponential Smoothing models, to determine their effectiveness in the Kenyan context.

The distinct characteristics of domestic and external debt in Kenya further justify this research. Each debt component responds differently to economic factors and policy interventions, requiring specialized analytical approaches. Current forecasting methods often treat public debt as a homogeneous entity, failing to account for these crucial differences. This study's comparative analysis of forecasting models for different debt components addresses this analytical shortcoming.

Additionally, the increasing complexity of Kenya's debt structure, coupled with global economic uncertainties, demands more sophisticated forecasting tools. Traditional forecasting methods have proven inadequate in capturing the dynamic nature of modern debt accumulation patterns. The evaluation of advanced time series models is therefore justified by the need for more robust analytical tools that can account for these complexities.

The absence of comprehensive model comparison studies in Kenya's debt forecasting literature also justifies this research. While individual forecasting methods have been applied in isolation, there has been limited systematic comparison of different approaches to determine their relative effectiveness. This study fills this gap by providing a rigorous comparative analysis of two prominent forecasting methodologies.

This research is further justified by the practical needs of policymakers and stakeholders who require reliable forecasting tools for effective debt management. The lack of validated forecasting models has hindered the development of evidence-based debt management strategies. By evaluating and comparing different forecasting approaches, this study addresses a critical need in Kenya's economic planning framework.

1.4 Research Objectives

1.4.1 General Objective

To model and forecast the Kenya's public debt using time series analysis.

1.4.2 Specific Objectives

1. To forecast Kenya's public debt using the ARIMA model.
2. To forecast Kenya's public debt using Holt Exponential Smoothing method
3. To compare the forecasting power of the ARIMA and the HES method and determine the perfect model for forecasting Kenya's public debt.

1.5 Research Questions

1. What are the forecasted values of Kenya's public debt using the Autoregressive Integrated Moving Average (ARIMA) model?
2. What are the forecasted values of Kenya's public debt using the Holt Exponential Smoothing (HES) method?
3. Which time series-forecasting model, the ARIMA or the Holt Exponential Smoothing method, exhibits superior performance in terms of accuracy and reliability for forecasting Kenya's public debt?

1.6 Scope of the study

This study focused on forecasting Kenya's public debt using monthly time series data spanning from January 2010 to December 2023. The research utilized secondary data obtained from the Central Bank of Kenya (CBK), comprising monthly observations of external debt, domestic debt, and total public debt figures. The study employed time series modeling algorithms, specifically the ARIMA and Holt Exponential Smoothing models, to analyze the public debt dataset. The scope was limited to these two forecasting methodologies and their application to Kenya's monthly debt data, with the aim of determining their relative effectiveness in predicting future debt patterns.

CHAPTER TWO

LITERATURE REVIEW

2.1 Empirical Review of the ARIMA and the Holt Exponential Smoothing (HES)

2.1.1 Autoregressive Integrated Moving Average (ARIMA)

The Autoregressive Integrated Moving Average (ARIMA) model has emerged as the preeminent univariate prediction model in time series analysis. Its distinctive capability to account for serial dependence among observations makes it particularly effective in scenarios where traditional exponential smoothing methods prove inadequate or inappropriate. The model encompasses a broad class of time series approaches, including autoregressive (AR) and moving average (MA) components, while effectively handling non-stationary data. The methodology, developed by Box and Jenkins (Montgomery et al., 2016), has become so fundamental to time series analysis that ARIMA is often referred to as the Box-Jenkins model in academic literature.

A significant advantage of the ARIMA model lies in its versatility regarding seasonality components. Numerous scholars have validated this flexibility, demonstrating its effectiveness in both seasonal and non-seasonal applications (Al Salman, 2015; Coutinho, 2012; Dramani & Frimpong, 2020; Lidiema, 2017; Montgomery et al., 2016; Mutwiri, 2019; Nahoussé, 2019). The model represents an extension of the ARMA methodology, specifically adapted for differenced series to handle non-stationary time series data commonly encountered in practical applications (Benvenuto et al., 2020; Zhao et al., 2020). The ARIMA model is conventionally denoted as ARIMA(p,d,q), where: p represents the order of the autoregressive model, d indicates the order of differencing required for stationarity, q denotes the order of the moving average model. The model can be expressed mathematically as:

$$\varphi(B)(1 - B)^d y_t = \theta(B)\varepsilon_t \quad (2.1)$$

Where: $\varphi(B) = B^p$ is the autoregressive polynomial of order p, $(1 - B)^d$ = differencing operator of order d, y_t is the time series variable, $\theta(B) = B^q$ is the moving average polynomial of order q, ε_t is the white noise error term. The ARIMA methodology follows a structured approach comprising four key phases: identification, estimation, diagnostic

checking, and forecasting. The identification phase primarily involves analyzing Autocorrelation Function (ACF) and Partial Autocorrelation Function (PACF) plots to determine appropriate p , d , and q values. Parameter estimation follows, typically employing Maximum Likelihood Estimation (MLE) or Conditional Least Squares Estimation (CLSE). Diagnostic checking assesses model fit and verifies whether residuals exhibit white noise characteristics. Once these conditions are satisfied, the model can be employed for forecasting.

The model's versatility extends across diverse fields, including economics, finance, engineering, and environmental science. This broad applicability stems from its ability to capture complex patterns in time series data. However, it's crucial to note that ARIMA modeling assumes linear processes and requires stationarity (either inherent or achieved through differencing). When these assumptions are violated, alternative models or transformations become necessary.

Recent applications of ARIMA modeling in economic forecasting have demonstrated its effectiveness. Muma and Karoki (2022) conducted a systematic review of ARIMA applications in GDP modeling, highlighting its efficiency and methodological robustness. In the context of fiscal forecasting, Micheni and Atitwa (2022) employed SARIMA and Holt-Winters models to predict Kenya's domestic tax revenues, establishing ARIMA's utility in fiscal planning.

The model's application in regional economic analysis has been particularly noteworthy. Samuel (2023) proposed a seasonal naive model with trend components for Kenyan tax revenue forecasting, complemented by ARIMA methodology. Maugu et al. (2023) examined public debt's impact on interest rates across East African Community member states, emphasizing the importance of accurate debt prediction for policy formulation. Mohamed (2022) successfully applied ARIMA models to forecast Somalia's economic growth, while Mbusi (2022) explored their utility in analyzing construction sector output and policy development in Kenya.

In the specific context of public debt forecasting, recent studies have demonstrated ARIMA's effectiveness. Razinkova et al. (2023) successfully employed ARIMA models to forecast Ukraine's external public debt during periods of uncertainty, highlighting the

model's stability in volatile environments. Sahib and Ibrahim (2022) utilized ARIMA modeling to analyze unemployment rates and economic growth relationships, demonstrating its versatility in labor market analysis.

Within the Kenyan context, Njenga (2024) applied ARIMA methodology to analyze domestic credit growth, providing valuable insights for public debt component forecasting. Muti and Opuodho (2023) examined correlations between macroeconomic factors and government bond yields listed on the Nairobi Securities Exchange, identifying crucial predictors for public debt analysis using ARIMA models. Further supporting the model's utility in macroeconomic forecasting, Ingale and Senan (2023) successfully applied ARIMA techniques for GDP analysis and forecasting.

The extensive track record of ARIMA modeling in various forecasting applications, coupled with its theoretical robustness, makes it particularly suitable for analyzing Kenya's public debt patterns. The model's ability to generate accurate forecasts provides essential information for policymakers, enabling evidence-based decision-making in debt management and economic growth strategies. Its success in handling both domestic and external debt components, while accounting for various economic factors, positions ARIMA as a valuable tool for comprehensive public debt analysis and forecasting.

The integration of ARIMA modeling in public debt forecasting represents a significant advancement in economic planning tools. Its ability to capture complex temporal dependencies, handle non-stationary data, and provide reliable forecasts makes it an indispensable instrument for fiscal policy formulation. The model's successful application across various economic contexts, particularly in developing economies similar to Kenya, further validates its selection for public debt analysis in this study.

2.1.2 Holt-Winters Exponential Smoothing (HES)

The Holt-Winters Exponential Smoothing method also well known by the name of the Triple Exponential Smoothing is the time series, forecast model which was developed by a person called Peter Winters (Habibur et al., 2016; Gelper et al., 2010). As a result, it is useful in the prediction of series including outliers and thus reduces the impacts of random fluctuations in time series (Box, et al., 1970; Habibur, et al., 2016; Munarsih & Saluza, 2020). The stochastic model has two parts namely base and trend which make it

capable of modelling and forecasting time series data that contains trend and level. The Holt Exponential Smoothing method uses weighted historical trends to predict future values and consists of two components: The Smoothing constant and the trend; The use of moving average (Lidiema, 2017; Makatjane & Moroke, 2016).

However, Holt Exponential Smoothing is rather accurate when it is being used with the singular purpose of predicting smoothly. It is useful because it can establish the trend and level within the time series data and therefore can be utilized to forecast additional shifts in the public debt in order that suitable action could be taken and policies created. Bolarinwa et al. (2022) compared the forecast behavior of four models, namely: In attempting to forecast on Nigeria's external reserves, the Holt Exponential Smoothing method was applied. This has further outlined the various strengths and weaknesses of each of the models, and how one can decide on the best method of forecasting in any given environment. Holt's Exponential Smoothing is trialed or utilized in other sectors and research such as economics, finance, energy forecast (Razinkova et al., 2023; Bolarinwa et al., 2022). This is advantageous while modelling and forecasting since it captures both the trend and the level in the data series and given that most of the macroeconomic variables such as the public debt are characterized by both trends and levels.

The aim of using the Holt Exponential Smoothing technique to forecast the Kenyan public debt is to forecast the future trend of the public debt in Kenya to aid policymakers and budget makers. These forecasts in one way can be of immense aid in determining the best way of handling the debt, can support the allocation of precious resources, and can support the planning and decision making process in the economic domain.

2.2 Comparison between the ARIMA and HES method

The issue of which of the two models is more suitable for forecasting is rather debated, and a number of researches have been aimed at identifying the efficiency of predictive abilities of those two models in various contexts. Depending on the properties of the data and the nature of the forecasts that are to be made, the either the model is opted. For instance, Zhao et al. (2020) employed both the ARIMA and HES if weather forecasting of the novel coronavirus in Wuhan, China. To conduct the comparisons MAPE was used and as seen in the result it established that the efficiency of the ARIMA model in terms of

forecasting was higher than that of the exponential smoothing technique due to the low percentage errors.

However, other researchers in other works could not confirm such findings. For instance, Makatjane and Moroke (2016) employed two models of forecasting namely ARIMA and triple exponential smoothing to compare the car sale in South Africa. Based on these results, they suggested that the exponential smoothing method has better forecast precision than the suggested ARIMA model. Sahinli (2020) has compared ARIMA model with the Holt Winters exponential smoothing method to arrive at which model is better for forecasting the prices of potatoes. The comparison of the results also showed that ARIMA had a smaller RMSE than the Holt Winters and therefore was better suited to forecast the prices of potatoes. Similarly, Munarsih and Saluza (2020) predicted the dengue fever cases, by employing the ARIMA model as well as the Holt-Winters exponential smoothing model. Concerning the accuracy measures, the table revealed that the percent errors were slightly lower for the ARIMA model than the percent errors for the ESM – Holt-Winters method, so the former was more accurate in the case of forecasting the dengue fever cases.

To see the comparison between the two models Holt-Winters exponential smoothing and the Box-Jenkins ARIMA model for the software failures, the readers can refer to work done by Yakovyna and Bachkai (2018). Subsequently, they were able to establish that the ARIMA model enabled them to make better prognosis of failed softwares than the Holt-Winters exponential method. However, Da Veiga et al., (2014) used two models to forecast the food demand in retail namely the ARIMA and the Holt-Winters model. According to the findings made by the authors, it was evident that the Holt-Winters model gave a better forecast of the demand of the foods than the ARIMA model. The findings above suggest that which one to use between the ARIMA and HES should be based on the characteristics of data to be analyzed as well as the requirement for forecasting. With the aim of achieving this, this paper will examine the level of accuracy of each of the selected models in the forecasting of Kenya's public debt using the root mean squared error (RMSE) measure.

In the following points of discussions, it is worthy to highlight that, using triple exponential smoothing and ARIMA models, Gathing (2014) estimated inflation in Kenya. Her work focused on giving the investors best predictions to encourage investment with reference to the current inflation rate and other adjustments. Thus, based on the analysis of the CIP, Gathing said that the optimal model for forecasting was the ARIMA model of order (1,1,0). While the actual and predicted values for the two models were close, the ARIMA model turned out to be the best prediction model for inflation in her study.

2.3. Modelling and Forecasting Public Debt

Public debt has been a topic of discussion for many scholars, which has brought about increased government spending due to high taxation thus improving the standard of living in many countries (Alloza et al., 2020; Andrés et al., 2017; Burriel et al., 2020; Leal et al., 2007). Barro (1979) held the view that public debt is developed when total income of the government through taxation and from other sources is unable to cater for expenditure. He said that constant income cannot support ever rising expenditure which is equal to total income mainly derived from taxes. As highlighted earlier, public debt forecasting is a very important issue, which has seen many works done to model and forecast the public debt globally. The analysis identifying the specificity of the application of the ARIMA model concerning the government debt prediction was conducted by Filatova and Aiyedogbon (2020) on the example of Ukraine. Two of them used the ARIMA model with the debt data set they analyzed the model provided reasonable estimates for the predictions suggesting that the model can be applied to other related studies.

Research on Kenya's public debt forecasting has employed various methodological approaches. Wanjiru (2020) conducted a comparative analysis using a hybrid neural network and ARIMA models to explain and predict the country's GDP and public debt. When evaluating model fit using RMSE, the ARIMA model demonstrated superior predictive accuracy compared to the hybrid neural network approach.

In a different study, William et al. (2019) utilized debt sustainability analysis to assess Kenya's future debt trajectory. Their findings suggested that without alternative funding mechanisms for the economy, Kenya's public debt would likely continue to rise.

These studies represent ongoing efforts to develop and evaluate public debt forecasting models, both internationally and specifically for Kenya. The work of Filatova and Aiyedogbon (2020) and Wanjiru (2020) particularly highlights the effectiveness of ARIMA models as valuable tools for government debt prediction. While alternative approaches like hybrid neural networks and debt sustainability analysis provide additional insights into public debt trends, there remains a need for further research to expand the literature on public debt forecasting, given its crucial role in economic stability and policy formulation.

The diverse modeling approaches documented by these researchers provide a foundation for governments, researchers, and decision-makers to develop forecasting systems that are tailored to their specific economic contexts and challenges, particularly in countries like Kenya.

2.4 Research Gap

The comparative analysis of ARIMA and HES models continues to be an active area of research across various fields, as evidenced by several recent studies (Zhao et al., 2020; Makatjane & Moroke, 2016; Şahinli, 2020; Munarsih & Saluza, 2020; Yakovyna & Bachkai, 2018; Da Veiga et al., 2014). A critical review of the literature reveals that the performance outcomes of these models are context-dependent and often yield ambiguous results across different applications. This context-specificity suggests that researchers must conduct comprehensive assessments to determine the most appropriate forecasting approach for Kenya's public debt, rather than assuming universal applicability of any single model.

While previous research has addressed the prediction of Kenya's GDP and public debt, notably Wanjiru's (2020) study employing ARIMA and HES models, there remains a significant gap in understanding how these models perform when applied specifically to the components of public debt, such as domestic and external debt. A more granular analysis of these debt elements could provide valuable insights into the models' predictive capabilities and offer a more nuanced understanding of Kenya's debt dynamics.

Furthermore, although some researchers have attempted to model and forecast Kenya's public debt (Wanjiru, 2020; William et al., 2019), the current literature review indicates a

scarcity of recent research in this domain. This gap is particularly concerning given the dynamic nature of economic factors and global public debt levels. The constant evolution of economic conditions, both domestically and internationally, necessitates frequent analysis to capture the current status of public debt accurately, enable policymakers to formulate appropriate and timely responses, account for emerging economic trends and challenges, and assess the impact of recent policy interventions.

The absence of recent comprehensive studies also limits our understanding of how various forecasting models perform under current economic conditions. This knowledge gap could potentially affect the accuracy of debt management strategies and policy formulation. Therefore, there is a pressing need for updated research that not only applies these forecasting models to Kenya's current economic context but also evaluates their effectiveness in predicting specific components of public debt. An additional research opportunity lies in examining how these models perform when accounting for recent global economic shifts and their impact on Kenya's debt profile. Such analysis would contribute to both the theoretical understanding of forecasting models and their practical application in public debt management.

CHAPTER THREE

RESEARCH METHODOLOGY

3.1 Data Source and Transformation

The study utilized historical public debt data obtained from the Central Bank of Kenya's (CBK) official website (<https://www.centralbank.go.ke>). The dataset comprises monthly observations of Kenya's public debt spanning from January 2010 to December 2021, encompassing 144 data points. The CBK portal provides open-access statistical data, including comprehensive records of both domestic and external debt figures, making it a reliable and authoritative source for public debt information. The raw data was downloaded in Excel format from the CBK's statistical database section.

After extraction, the dataset underwent several preprocessing steps using R statistical software (version 4.1.2) to prepare it for analysis. The transformation process included data cleaning to address any missing values, inconsistencies, or data entry errors in the raw dataset. Subsequently, the time series was structured by properly formatting the date variables and ensuring chronological ordering of the observations.

Exploratory Data Analysis (EDA) was conducted using time series plots to visually inspect the data for underlying trends (upward, downward, or cyclic patterns), seasonal patterns or periodic fluctuations, potential outliers or anomalous observations, and structural breaks or regime changes in the series. This visual inspection provided crucial insights into the characteristics of the public debt data and guided the selection of appropriate analytical techniques.

Statistical transformations were applied to ensure the data met the assumptions required for time series modeling. These transformations included smoothing techniques to reduce noise in the series, tests for stationarity were performed, and where necessary, differencing was applied to achieve stationarity. Additionally, seasonal adjustments were made if seasonal patterns were detected. These preprocessing steps were essential to prepare the data for subsequent modeling and analysis, ensuring the reliability and accuracy of the forecasting results.

3.2 Model Fitting and Selection

The selection of the Akaike Information Criterion (AIC) as the model selection metric was deliberate and theoretically justified. While other information criteria like the Schwarz Bayesian Information Criterion (SBIC) and Hannan-Quinn Information Criterion (HQIC) are also valid options for model selection, AIC offers specific advantages that align with the objectives of this study.

AIC was preferred because it strikes a balanced trade-off between model accuracy and complexity, particularly suitable for time series forecasting. Unlike SBIC and HQIC which impose stronger penalties for model complexity, AIC's relatively lighter penalty term makes it more appropriate when the true data-generating process might be complex, as is often the case with public debt dynamics. This characteristic of AIC helps retain important predictor variables that might be excluded by the stricter criteria of SBIC or HQIC, potentially leading to better out-of-sample forecasting performance.

Furthermore, in small to moderate sample sizes, which is the case with our monthly debt data spanning 2010-2021, AIC tends to perform well in selecting models that capture the underlying data patterns without overfitting. While SBIC and HQIC are known to be more consistent in selecting the true model as sample size approaches infinity, our finite sample makes AIC's efficiency properties more relevant. The focus on forecast accuracy rather than model parsimony in this study also aligns with AIC's properties, as it tends to select models that minimize prediction error, which is crucial for debt forecasting applications.

3.3. Model Specification

3.3.1 Autoregressive Integrated Moving Average Model

According to Box et al. (1970) and Montgomery et al. (2008), Box and Jenkins formerly known as the Box-Jenkins methodology first proposed the ARIMA model. ARIMA model uses past information and error terms when making forecasts. The ARIMA model is made up of three components i.e., the differenced term (d), the Moving Average (MA) term, and the Autoregressive (AR) term. The longer the historical information, the better the prediction as the model progresses. Stationarity is a requirement for model fitting (Kinyili & Wanyonyi, 2021; Gechore et al., 2022). The ARIMA model is a type of ARMA model that includes at least one different term.

ARIMA (p, d, q) model is thus given by $\nabla^d y_t = (1 - B)^d y_t$

Where, $\nabla = 1 - B$ and B is the backward shift operator. The *ARIMA* model is generally written as.

$$\alpha_p (B)(1 - B)^d y_t = \beta_q (1 - B)\varepsilon_t \quad (3.1)$$

A first-order differenced public debt series can be expressed in the form:

$$\Delta PD_t = (PD_t - PD_{t-1}) \quad (3.2)$$

Where PD_t is the public debt at time t . Therefore, the *ARIMA* ($p, 1, q$) model is expressed as

$$PD_t = \beta_0 + \phi_1 PD_{t-1} + \dots + \phi_p PD_{t-p} + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \dots + \theta_q \varepsilon_{t-q} \quad (3.3)$$

The above equation (3.3) must first be stationary, otherwise, successive differencing is conducted till this condition is met. The aim is to find the most appropriate *ARIMA* (p, d, q) model that can be used for prediction.

The *ARIMA* model implementation requires several key assumptions to be satisfied to ensure reliable forecasting results. One of the fundamental assumptions is stationarity, which necessitates that the time series data maintains consistent statistical properties, including mean, variance, and autocorrelation, throughout the observed period. Analysis of the original public debt series revealed non-stationary characteristics, necessitating two rounds of differencing to achieve the required stationarity before proceeding with the *ARIMA* model fitting.

Another critical assumption underlying *ARIMA* modeling is the presence of linear relationships between the current value of the time series and its historical values, as well as between current and past forecast errors. This linearity assumption forms the basis for the model's ability to capture and project patterns in the data effectively. The model further assumes that the residuals, which represent the differences between the actual observed values and the model's fitted values, follow a normal distribution pattern, enabling statistical inference and confidence interval estimation.

Additionally, the ARIMA framework requires independence among the residuals, meaning there should be no significant autocorrelation between successive residual terms. This independence assumption is crucial as it ensures that the model has effectively captured all meaningful patterns in the data, leaving only random, uncorrelated noise in the residuals. These assumptions collectively form the theoretical foundation for ARIMA modeling and must be validated through appropriate diagnostic tests to ensure the model's reliability and accuracy in forecasting public debt trends.

3.3.2 Adaptive Holt-Winters Forecasting Model

This study implements an adaptive form of Holt-Winters exponential smoothing, a flexible time series prediction technique introduced by Charles Holt in 1957 (Qader et al., 2021; Siddiqui et al., 2022; Cifonelli & Canu, 2022). The methodology models data by decomposing it into evolving level and trend components and uses weighted averaging to forecast future values.

The level represents the current series baseline, while the exponential trend captures the general direction and rate of change. The model assigns relative weights between past and recent data points to optimally balance responsiveness and smoothing. These data-driven smoothing constants dynamically adapt based on autocorrelations at each timestep rather than using fixed empirically chosen values.

Critically, this adaptive approach accommodates intrinsic time series non-stationarity, where the underlying statistical properties shift gradually over time. This contrasts with most classical techniques that assume constancy. The evolving Holt-Winters framework circumvents this limitation by recursively tracking and projecting changes as they emerge. The Adaptive Holt-Winters Forecasting methodology is expressed as;

$$\text{Level: } L_t = \alpha Y_t + (1 - \alpha)(L_{t-1} + T_{t-1}) \quad (3.4)$$

$$\text{Trend: } T_t = \beta(L_t - L_{t-1}) + (1 - \beta)T_{t-1} \quad (3.5)$$

$$\text{Forecast: } F_{t+h} = L_t + hT_t \quad (3.6)$$

Where Y_t is the observed value at time t , L_t is the level at time t , T_t is the trend at time t , α is the smoothing parameter for the level, with $0 < \alpha < 1$, β is the smoothing parameter

for the trend, with $0 < \beta < 1$, and h is the forecast horizon. Smoothing parameters α and β are constant that define the proportion of current observation to the past observation

The Holt-Winters exponential smoothing model has a different set of assumptions:

Level, Trend, and Seasonality: The Holt-Winters method requires the time series to be writable as a sum of an evolving level, trend, and seasonal term.

Adaptive Smoothing: The Holt-Winters approach uses three smoothing parameters that are adaptive meaning that they change with time and with the data being analyzed.

No Residual Autocorrelation: Even though the Holt-Winters model does not impose the requirement of residuals to be normally distributed, it does so implicitly for the purpose of forecasting the next value, by assuming that the residuals are independent and not auto correlated.

3.4 Diagnostic Test

The validation of the selected model's conformity to stationarity specifications involved a comprehensive testing process. Following the model fitting and parameter estimation phases, rigorous diagnostic checks were performed on the fitted model. As outlined by Maurice et al. (2021) and Gechore et al. (2022), model validation in time series analysis is particularly effective when parameters are estimated using the maximum likelihood estimation technique.

The assessment of model normality employed both normal Q-Q plots and histograms of residuals, which demonstrated the expected bell-shaped distribution characteristic of normally distributed data. To evaluate the independence of residuals, the Ljung-Box test statistic was employed. This test examined the null hypothesis that no serial correlation exists between the lags, thereby testing whether the residuals are independent and identically distributed. This systematic approach to model validation ensured the reliability and statistical validity of the forecasting model, providing confidence in its ability to generate accurate predictions of public debt trends.

The integration of these diagnostic tools and statistical tests provided a comprehensive framework for assessing the model's adherence to key assumptions, ultimately validating its suitability for the intended forecasting application.

3.5 Comparative Evaluation

To determine which of the two models was the most accurate at forecasting Kenya's future debt, both the chosen ARIMA and Holt models were evaluated on out-of-sample predictive performance. The actual historical debt data starting from 2010 to 2021 was applied in developing the two models and then used in coming up with the forecast for 2022 to 2023. These debt forecasts were compared against the actual debt figures from 2022-2023 recently released by the Central Bank of Kenya. The Root Mean Square Error (RMSE) was also applied for model comparison. Between the two models i.e., the model with the least errors was considered an optimal model for forecasting Kenya's public debt projections.

This rigorous predictive validity assessment focuses specifically on the models' abilities to forecast multiple future years in Kenya's debt time series. By comparing each model's multi-year forecasts to the true realized debt figures, the accuracy and reliability of the ARIMA and Holt models can be evaluated. Examining performance on this longer-range out-of-sample forecasting provides a robust evaluation for determining the preferable model for generating Kenya's future debt estimates. The model generating forecasts with the lowest deviation from the actual debt provides higher confidence in its applicability for ongoing estimation of Kenya's debt trajectory.

CHAPTER FOUR

RESULTS AND DISCUSSION

4.1 Summary Statistics

The dataset in Table 4.1 contains 144 observations for each variable (domestic debt, external debt, and total debt), with no missing values. This indicates a complete dataset, allowing for a comprehensive analysis without the need for missing data handling techniques. The means provide an estimate of the central or typical values, with domestic debt at 1.853, external debt at 1.863, and total debt at 3.715. The medians, which are less influenced by extreme values, are slightly lower than the means, suggesting some positive skewness in the distributions.

The standard deviations measure the spread or dispersion of the data around the mean. Domestic debt has the smallest standard deviation (0.993), indicating less variability compared to external debt (1.132) and total debt (2.123). The larger standard deviation for total debt suggests a wider spread of values around the mean. The minimum and maximum values indicate the range of the data, with domestic debt ranging from 0.581 to 4.032, external debt ranging from 0.525 to 4.174, and total debt ranging from 1.106 to 8.207. These ranges provide an understanding of the lowest and highest values observed in the dataset.

The first and third quartiles (25th and 75th percentiles, respectively) provide information about the distribution of the data. For domestic debt, 25% of the values are below 0.968, and 75% are below 2.573. For external debt, 25% of the values are below 0.816, and 75% are below 2.712. For total debt, 25% of the values are below 1.791, and 75% are below 5.288.

The positive skewness values for all three variables (domestic debt: 0.570, external debt: 0.520, total debt: 0.545) indicate that the distributions are skewed to the right, with longer tails on the right side. This suggests the presence of some larger values or outliers in the data. The negative kurtosis values for all three variables (domestic debt: -0.911, external debt: -1.088, total debt: -1.010) indicate that the distributions are relatively flat or platykurtic, with fewer extreme values in the tails compared to a normal distribution.

From the summary statistics, it is possible to get the general picture of the characteristics of the domestic debt, external debt, and total debt data. The information about central tendency, variability, ranges, quartiles and distribution shapes can help to get some insight to the data and to decide whether there can be outliers or skewness in data and how to proceed with the further analysis or with choosing the proper statistical method or model.

Table 4.1: Summary Statistics for Domestic Debt, External Debt, and Total Debt

Statistic	Domestic Debt	External Debt	Total Debt
n	144	144	144
Missing	0	0	0
Minimum	0.581	0.525	1.106
Maximum	4.032	4.174	8.207
1st Quartile	0.968	0.816	1.791
3rd Quartile	2.573	2.712	5.288
Mean	1.853	1.863	3.715
Median	1.532	1.631	3.167
Sum	266.775	268.201	534.976
SE Mean	0.083	0.094	0.177
LCL Mean	1.689	1.676	3.365
UCL Mean	2.016	2.049	4.065
Variance	0.986	1.282	4.507
SD	0.993	1.132	2.123
Skewness	0.570	0.520	0.545
Kurtosis	-0.911	-1.088	-1.010

SE Mean = Standard Error of the Mean; LCL Mean = Lower Confidence Limit for the Mean; UCL Mean = Upper Confidence Limit for the Mean; SD = Standard Deviation.

4.2 Time series Plots

The time series plot of domestic debt in Kenya as shown in Figure 4.1 evidences this over the past decade, which has been on the rise, thus showing an increase in the borrowing from the domestic sources by the Kenyan government. When it comes to domestic debt the graph indicates it has increased from 0.58 trillion KES in January 2010 to 4 trillion KES in December 2021. Although, there exist slight fluctuations in the graph, the general trend depicts an upward bowl; that is, there is an increasing tendency in the use of domestic borrowing to fund government expenditures.

The time series plot of the external debt in Kenya also rises, though with higher volatility compared to the domestic debt. International liabilities have risen from about 0.53 trillion KES in January 2010 to almost 4.17 trillion KES by December 2021. From the plot, one is also able to identify periods of growth that are higher, for instance between 2014 and 2015 and other periods that show relative stability or lower growth. These can be attributed to variations in borrowing, repayment or external borrowing and other related economic factors.

The graph for the national public debt comprising domestic and external debt reveals that there is a continuous increase in the total amount of money borrowed in the last two decades by the Kenyan Government. The current total public debt has increased from about 1.11 trillion Kenyan shillings in January 2010 to slightly over 8.2 trillion Kenyan shillings in December 2021. The trend line is characterized by gradual increase in the values, and the rate of increase seems to be rising in the recent period. This trend portrays the fact that Kenya's public debt has become a growing concern and its sustainability in the economy is questionable in the long run.

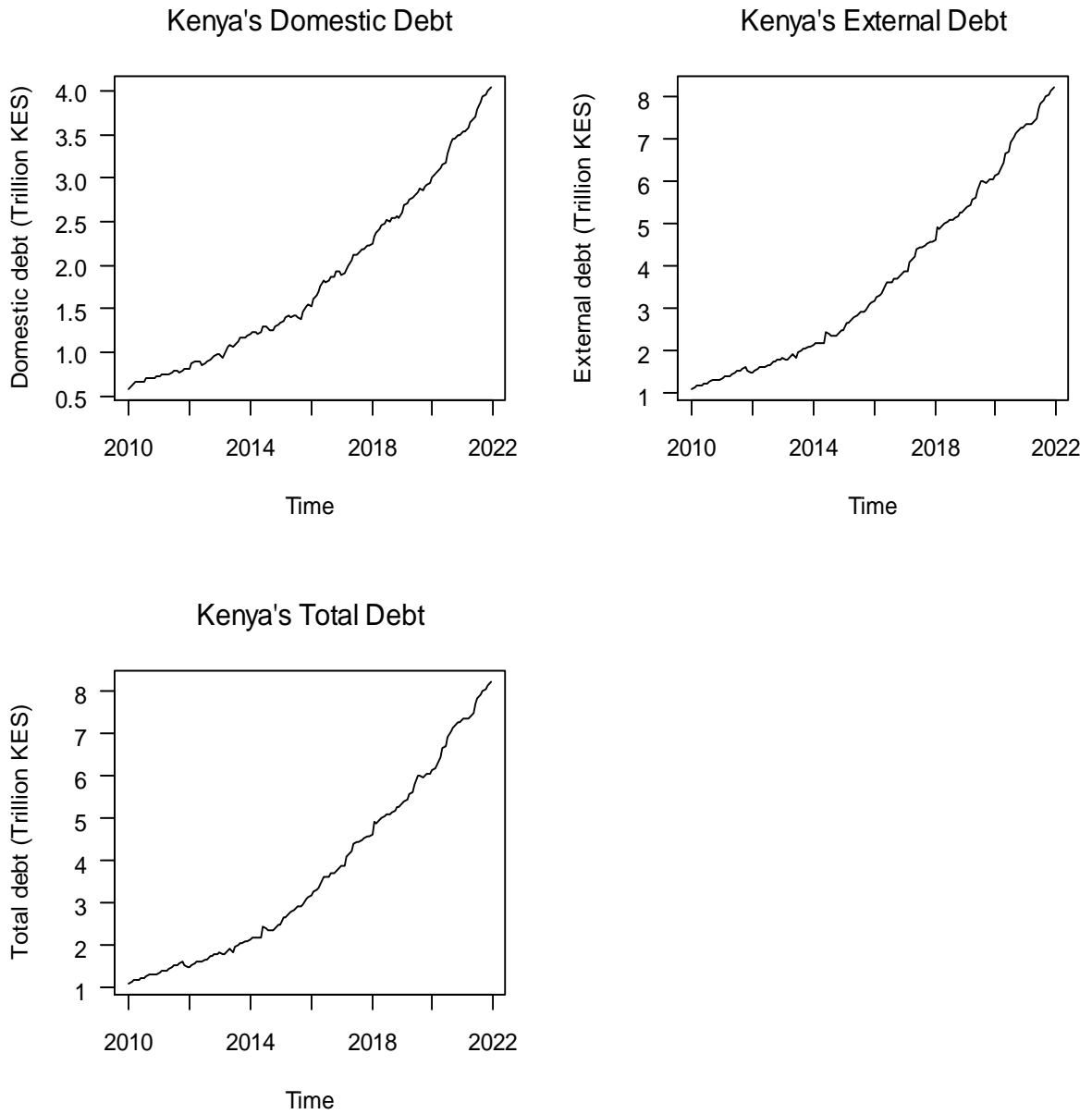


Figure 4.1: Time series plots

4.3 Stationarity Test

In time series analysis, the Augmented Dickey-Fuller (ADF) test is one of the most widely used statistical tests for the stationarity of the series. The test focuses on the unit root in the time series data and usually if the p-value is smaller than 0.05 the series is considered stationary. The results of the ADF test for domestic, external, and total debt are presented in Table 4.2:

Based on the p-value calculated in the table 4.2 it can therefore be concluded that the calculated p-values of domestic debt equal to 0.99; external debt = 0.8292; and total debt = 0.9667 are greater than the conventional significance level of 0.05. This implies that the source null hypothesis that all the three time series variables are non-stationary cannot be rejected.

From the findings, it is evident that the p-values and results of the test statistics are negative, making it clear that the time series data for domestic debt, external debt and total debt are non-stationary. Non-stationarity is a concept which conveys a theme around the time series, which means that mean and variance of the considered series may vary overtime.

Table 4.2: Augmented Dickey-Fuller Test Results for Stationarity

Variable	Test Statistic	Lag Order	p-value	Stationarity
Domestic Debt	0.21219	20	0.99	Non-stationary
External Debt	-1.393	20	0.8292	Non-stationary
Total Debt	-0.71817	20	0.9667	Non-stationary

4.4 Differenced Time Series

The findings of the ADF test indicated that the time series data on domestic debt, external debt, and total debt was non-stationary; thus, the data was subsequently differentiated twice in an attempt to deal with the issue. Another technique, which is often used when dealing with time-related series, is known as differencing and its purpose is to remove trends that make a series non-stationary (Verma et al., 2021; Salles et al., 2019).

The first square in the differencing was achieved by simply deleting every observation in the original time series and undertaking the subtraction of every observation with the preceding observation. However, when analyzing the result of the first-differenced data with the ADF test, it was identified that the series is remains non-stationary. Thus, after first-differencing, the second-order differencing was done and involved calculation of the difference of the first-differenced values.

Subsequently, the ADF test was conducted on the twice-differenced time series data after the second-order differencing. The ADF test statistics were significant and the p-values were less than the specified significance level such as 0.05 hence the twice-differenced series for domestic debt, external debt and total debt were stationary.

The second-order differencing makes the domestic debt, external debt and total debt time series stationary, which means that the statistical properties of the differences such as mean and variance are constant over time as presented in figure 4.2. This is a very important assumption for many of the procedures used in time series analysis such as the ARIMA and other forecast models.

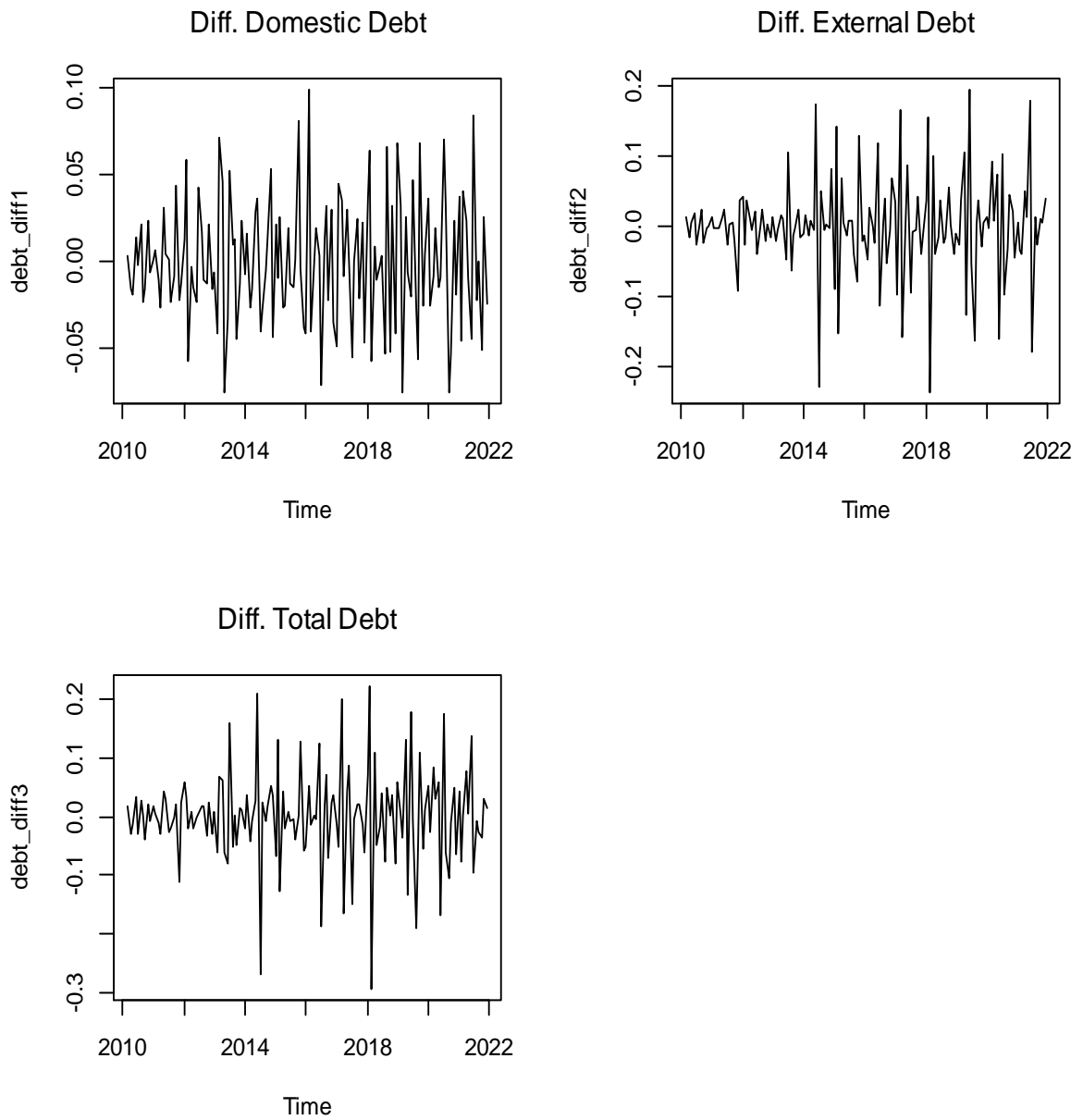


Figure 4.2: Differenced Time-series plots

4.5 ARIMA Model

4.5.1 ARIMA model Fitting

The model selection stage included choosing the best set of parameters for AR, I and MA that would best describe the temporal series.

For domestic debt, the best model selected is ARIMA(0,2,3)(0,0,1)[12], which suggests that model is non-seasonal, has no auto-regressive term, has a second order difference, third order moving average, and a first order moving average with a periodicity of 12 (probably a monthly or quarterly data). The model selection was performed using Akaike Information Criterion (AIC) which compares the models according to the balance between the fit to the data and the complexity of the model.

For the external debt, the chosen model was the ARIMA (0,2,1), which does not contain any of the auto regressive terms, the data are differenced twice and it contains one moving average term. There were no dummy variables for the seasonal variation in this model.

As for total debt, the selected model was the same as in case of the external debt – the ARIMA (0,2,1) model which points to the same pattern as the one described above. We found one estimate for the first order of moving average which is -0.9428 with a standard error of 0.0222. The value of the residual variance estimated for the model is 3.33×10^9 , which implies that the model can explain a considerable part of the variability in the data. The value of the global log-likelihood, which estimates the model's general fit, is -1758.84. The AIC score of 3521.69, which indicates a good fitness of the model with the differenced time series data.

4.5.2 Accuracy measures of the selected ARIMA model

The accuracy measures of the ARIMA models fitted on the domestic, external and total debt are summarized below in the following tables.

4.5.2.1 Domestic Debt

The ARIMA model for domestic debt shows a good fit to the data as indicated by the low values of the error measures. The RMSE of 0.02649721, reveals that the model's predictions are on average, off by about 0.026 trillion KES from actual domestic debt values. The MAPE of 1.313592% as a sign that the percentage error in the predictions is, on average, small as shown in Table 4.3.

Table 4.3: Accuracy Measures for ARIMA Model on Domestic Debt

Measure	Value
ME	0.002815098
RMSE	0.02649721
MAE	0.02036044
MPE	0.07177449
MAPE	1.313592
MASE	0.07213811
ACF1	-0.001151398

4.5.2.2 External Debt

The accuracy statistics of the ARIMA model on external debt are slightly lower, and therefore the model fits the data slightly less well. The RMSE of 0.04805216 reveals that the distance between the actual values of the external debts and the model's forecasts is approximately 0.048 trillion KES on average. The MAPE of 1.926205% is higher than that for domestic debt, which means that the average percentage error in the predictions is even greater as shown in Table 4.4.

Table 4.4: Accuracy Measures for ARIMA Model on External Debt

Measure	Value
ME	0.005339308
RMSE	0.04805216
MAE	0.03178391
MPE	0.3004599
MAPE	1.926205
MASE	0.1029995
ACF1	-0.07100679

4.5.2.3 Total Debt

The accuracy measures for the ARIMA model on total debt are somewhere in between the domestic and external debt measures, due to the combined results of the model on the two components. The RMSE of 0.05710133 indicates that the model's predictions are, on average, within about 0.057 trillion KES of the actual total debt values, which is higher compared with the RMSE for domestic debt, but lower than that for external debt as shown in Table 4.5.

Table 4.5: Accuracy Measures for ARIMA Model on Total Debt

Measure	Value
ME	0.00683816
RMSE	0.05710133
MAE	0.0382825
MPE	0.1550043
MAPE	1.173602
MASE	0.06479658
ACF1	-0.01594005

4.5.3 Time series Forecasting plots

The time series forecast plots for domestic, external, and total debt show an upward trend over the 27-month period from January 2022 to March 2024.

For domestic debt, the point forecast values in Table 4.6 start at 4.068896 trillion KES in January 2022 and increase steadily to reach 5.151958 trillion KES by March 2024. The upward trend is consistent throughout the forecast period, indicating a continued rise in domestic borrowing by the Kenyan government.

The external debt forecast exhibits a similar upward trajectory, starting at 4.210887 trillion KES in January 2022 and rising to 5.169948 trillion KES by March 2024. The rate of increase appears to be slightly steeper than that of domestic debt, suggesting a potentially higher reliance on external borrowing during the forecast period. This is shown in Figure 4.3

The forecast for total debt, which combines domestic and external debt, also shows a clear upward trend. The point forecast value in this case begins at 8.286081 trillion KES in January 2022 and reaches 10.342175 trillion KES in March 2024. This is equal to a projected increase in the overall public debt level over the 27-month period.

The forecast plots of domestic, external, and total debt show directions that are in harmony with the existing trends of increasing public debts in Kenya. These trends may be attributed to various factors for instance, the need to increase government expenditure, economic problems, or bill up of previous debts. However, it should be noted that such forecasts are based on the conditions and trends of the historical data whereas the actual future levels of debt could be affected by various factors such as economic conditions, changes in policies, etc.

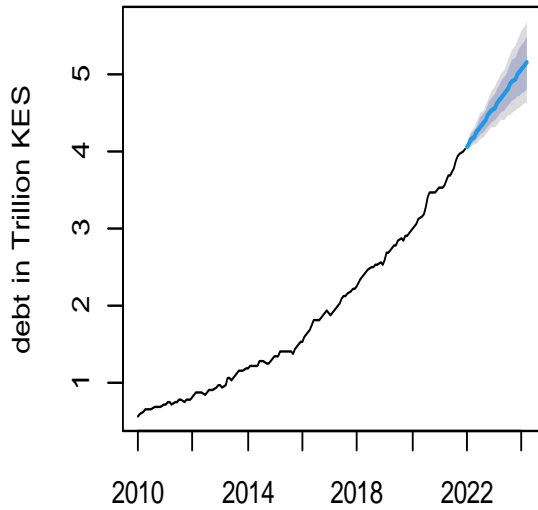
Table 4.6: Point Forecast Values for Domestic, External, and Total Debt

Month/Year	Domestic Debt	External Debt	Total Debt
Jan 2022	4.068896	4.210887	8.286081
Feb 2022	4.106525	4.247774	8.365161
Mar 2022	4.147701	4.284661	8.444242
Apr 2022	4.192148	4.321548	8.523322
May 2022	4.235560	4.358435	8.602403
Jun 2022	4.272427	4.395322	8.681483
Jul 2022	4.320727	4.432209	8.760564
Aug 2022	4.364857	4.469096	8.839644
Sep 2022	4.410488	4.505983	8.918725
Oct 2022	4.449863	4.542870	8.997805
Nov 2022	4.492530	4.579757	9.076886
Dec 2022	4.532026	4.616644	9.155966
Jan 2023	4.572701	4.653531	9.235047
Feb 2023	4.614448	4.690418	9.314128
Mar 2023	4.655795	4.727305	9.393208

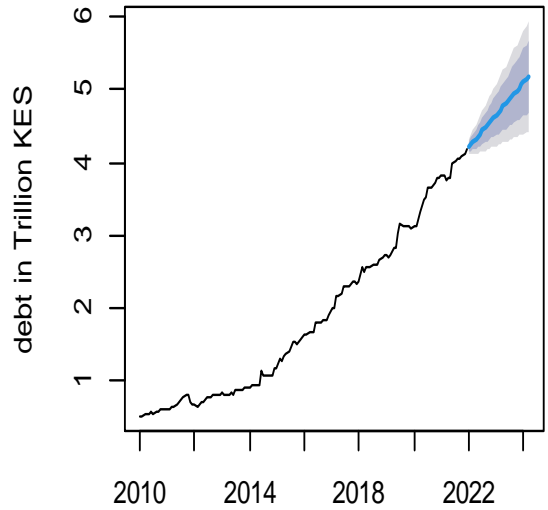
Apr 2023	4.697142	4.764192	9.472289
May 2023	4.738489	4.801079	9.551369
Jun 2023	4.779836	4.837966	9.630450
Jul 2023	4.821183	4.874853	9.709530
Aug 2023	4.862530	4.911740	9.788611
Sep 2023	4.903877	4.948627	9.867691
Oct 2023	4.945224	4.985514	9.946772
Nov 2023	4.986571	5.022401	10.025852
Dec 2023	5.027918	5.059288	10.104933
Jan 2024	5.069265	5.096175	10.184013
Feb 2024	5.110611	5.133062	10.263094
Mar 2024	5.151958	5.169948	10.342175

4.5.4 Forecasting plot for ARIMA

ARIMA Forecast for Domestic Debt



ARIMA Forecast for External Debt



ARIMA Forecast for Total Debt

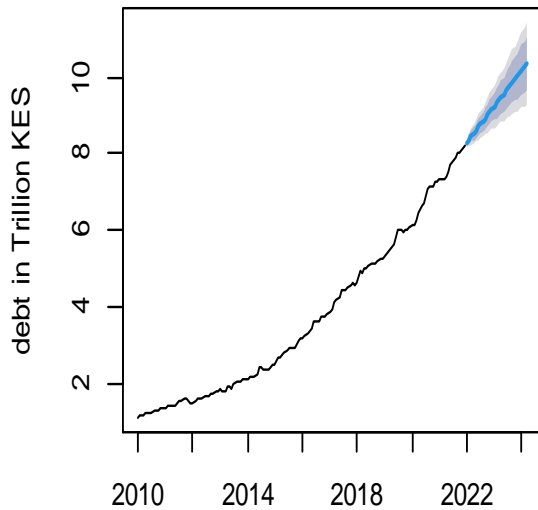


Figure 4.3: ARIMA forecasts

4.5.5 Diagnostic Test for ARIMA

The diagnostic plots of the ARIMA models in Figure 4.4 gives the relevant information about the adequacy of the fitted models. The histograms of the residuals for all the three series, namely, the domestic, external, and total debt are bell-shaped or can be said to follow an approximately normal distribution. This desirable characteristic indicates that the residuals are centered on zero, which is the basic assumption of the ARIMA model.

In addition, the normal Q-Q plots also support the conclusion made earlier on normality of residuals. In these plots, the residuals are perfectly on the diagonal line, so they are approximately normally distributed. The points on the Q-Q plots are also close to the diagonal line, which also supports the assumption of normality of the residuals.

The bell-shaped histograms and the residuals, which are placed on the diagonal line in the normal Q-Q plots, are the positive signs that the data in the ARIMA models has been captured appropriately and the underlying patterns have been discovered. In addition, these diagnostic plots indicate that the residuals are normal and independent, which are important assumptions in the case of the ARIMA models.

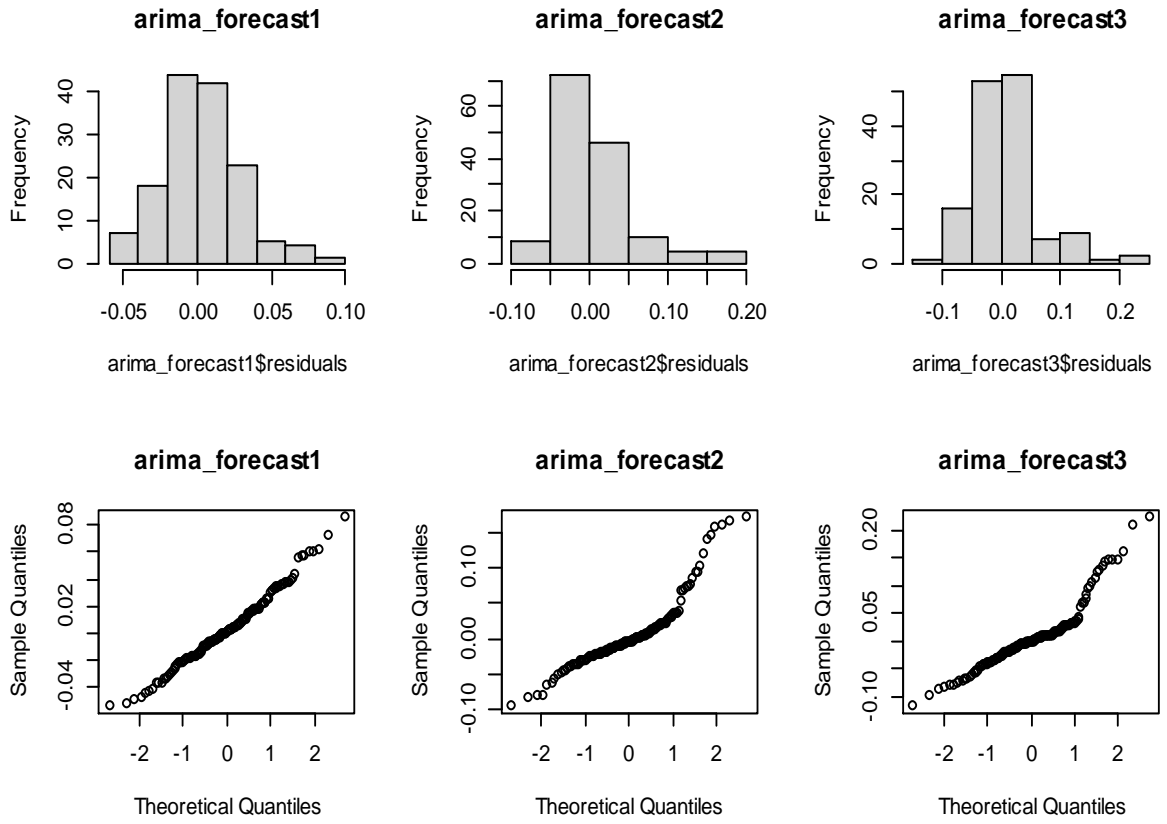


Figure 4.4: ARIMA diagnostic tests

4.6 Holt-Winters model Fitting and Selection

In this research, the Holt-Winters exponential smoothing model was used for the prediction of the domestic debt, external debt, and total public debt time series in Kenya. This model is useful for data that has both a trend and seasonality component. It employs the weighted moving average to adjust the level, trend, and season factors over time and is therefore capable of capturing changes in the series' behavior.

For the domestic debt series, the values of alpha, beta, and gamma were estimated as 1, 0.05163863 and 0.08238391 respectively. Alpha regulates the impact of the most recent observation on the current level estimate relative to the smoothed level estimate of the previous period on the level component. Beta determines how rapidly the trend component responds to changes in the trend of the data. Gamma governs the rate at which seasonal factors are updated from the seasonal patterns in the data.

The initial level was estimated at 4.0322951389 with an initial trend of 0.0401532905. The seasonal indices ranged from -0.0147534722 to 0.0122465278, capturing the fluctuations across the 12 months or periods within each yearly cycle. These coefficients define the starting values and seasonal adjustment factors used by the model.

For external debt, the smoothing parameters were alpha = 0.9081523, beta = 0.03533579, and gamma = 0.1924386. The initial level was 4.177148894 with a trend of 0.036764695. Seasonal indices ranged from -0.015101739 to 0.031526621 over the 12 periods.

The total public debt series had smoothing parameters alpha = 0.9443541, beta = 0.05346503, and gamma = 0.2541283. Its initial level was 8.2146691392 with an upward initial trend of 0.0790563057. Seasonal indices varied between -0.0251565909 and 0.0281980673 over the year.

The model fitting process to minimize the sum of squared forecast errors over the calibration period optimizes these smoothing parameters and initial values. The level smoothing (alpha) controls how rapidly the level component is updated towards the latest observation. The trend-smoothing (beta) governs the responsiveness of the trend estimate to recent changes. Seasonal smoothing (gamma) determines the weight of the current season's value versus past seasonal estimates.

The seasonal indices quantify the recurring monthly/quarterly fluctuations around the trend-cycle, allowing the model to account for intra-year seasonality in the debt series. Positive seasonal indices indicate periods with values above the trend-cycle, while negative indices correspond to periods below the trend-cycle.

The Holt-Winters model can accurately predict the future values, as it incorporates both the trend and seasonality into its calculations and can predict the future values based on the growth/decay patterns and cyclic variations. This makes it useful in applications where the data has such characteristics like public debt, which can be affected by business cycles, policies changes or seasonality in the course of time.

4.6.1 Accuracy measures for Holt-Winters models

The accuracy measures given for the Holt-Winters model in Table 4.7, give us an idea of how good the model is in the forecasting of domestic, external and total debt. For the Holt-Winters model, ME values are positive for all the types of debt suggesting that this model has a tendency of over-forecasting the actual values. The ME for domestic debt is less than for the external and total debt indicating that the average over-estimate for domestic debt is relatively low.

If we are looking at the Root Mean Squared Error or RMSE, which quantifies the mean magnitude of the forecast errors, the performance of the Holt Winter's Model is moderate. The RMSE for domestic debt is higher than the ARIMA model, thus, we can infer that the model has a poorer fit. Nonetheless, the Holt-Winters model is slightly better when it comes to external debt as the RMSE is slightly lower than that of the ARIMA model. However, for total debt, the RMSE of the Holt-Winters model is greater than that of the ARIMA model meaning that the model has a poor ability to forecast the total debt.

The Mean Absolute Error (MAE) which gives the mean of the absolute differences between the fitted and the actual values reveal that the Holt-Winters model has a relatively higher MAE than the ARIMA model for all types of debts, which means that the model yields comparatively larger absolute errors on an average. Also, the percentage error represented by the Mean Absolute Percentage Error (MAPE) shows that the Holt-Winters model has higher MAPE values for the domestic and total debts, which means that the model has relatively higher errors. On the same note, the MAPE for the external debt is

lower for the Holt-Winters model than for the ARIMA model, thus showing that it is relatively more accurate.

The Mean Absolute Scaled Error, which is a scaled version of the MAE also indicates that the Holt-Winters model has higher MASE values than the ARIMA model for all the debt types implying larger scaled errors. Also, the measure of Autocorrelation of errors at lag 1 commonly known as ACF1 that depicts any amount of autocorrelation left in the residuals is mixed. For the domestic debt, the forecast accuracy of the Holt-Winters model is slightly better than that of the ARIMA model as indicated by the higher value of ACF1 meaning that there is still some degree of autocorrelation in the residuals. But, for external and total debt, the ACF1 values of the Holt-Winters model are lesser than that of the ARIMA model, suggesting lesser remaining autocorrelation.

Table 4.7: Holt-Winters Accuracy measures

Debt Type	Model	ME	RMSE	MAE	MPE	MAPE	MASE	ACF1
Domestic	ARIMA	0.0028	0.0265	0.0204	0.0718	1.3136	0.0721	-0.0012
	Holt	0.0047	0.0311	0.0240	0.2444	1.4821	0.0850	0.0679
External	ARIMA	0.0053	0.0481	0.0318	0.3005	1.9262	0.1030	-0.0710
	Holt	0.0056	0.0494	0.0342	0.2468	2.0226	0.1108	-0.0083
Total	ARIMA	0.0068	0.0571	0.0383	0.1550	1.1736	0.0648	-0.0159
	Holt	0.0087	0.0614	0.0436	0.2275	1.3300	0.0739	-0.0144

4.6.2 Holt-Winters Forecast

In the case of domestic debt, the projection shows an upward trend from the year 2000 to 2008. The point forecast for the domestic debt rises to 4,073.070 trillion KES in January 2022 and reaches 5,127.264 trillion KES in March 2024 as shown in Table 4.8. This development indicates that there is always a dependence on domestic borrowing to fund government expenditures. In the same way, the projection of the external debt has a similar trend of rising in the future. While the point forecast of external debt increases from 4,208,626,000,000 KES in January 2022 up to 5,158,177,000,000 KES in March 2024.

This rise also suggests that the country is increasingly turning to external borrowings, which may raise questions regarding sustainability of debt and foreign exchange risk.

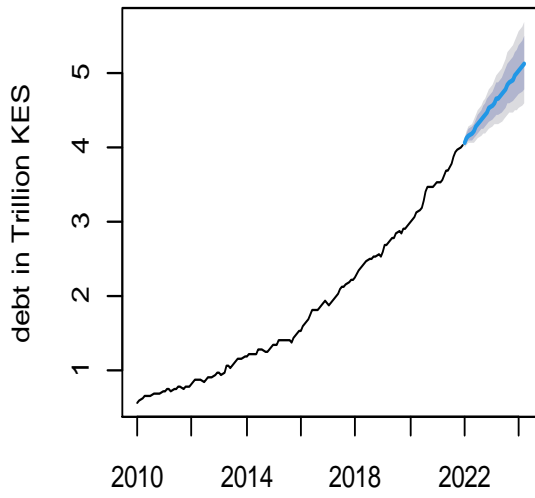
Therefore, the trend of the total debt, which is the sum of the domestic and external debt, is on the rise as shown in figure 4.5. The point forecast of total debt increases from 8,289,019 million KES in January 2022 to 10,349,294 million KES in March 2024. This upward trend points to the fact that the government's debt continues to rise, which may require proper balancing of the debt and fiscal policies.

Table 4.8: Holt-Winters point forecast

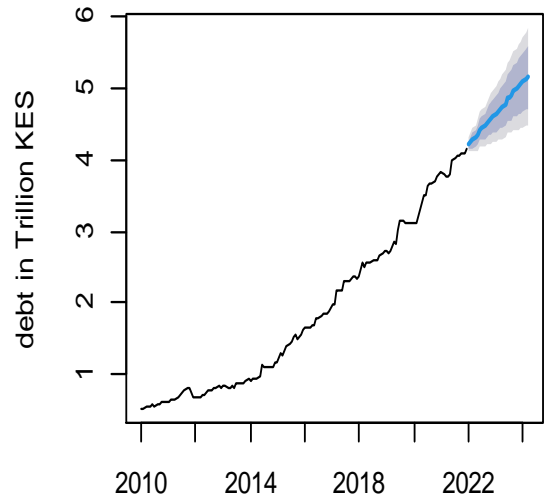
Year/Month	Domestic Debt (Trillion KES)	External Debt (Trillion KES)	Total Debt (Trillion KES)
Jan 2022	4.073070	4.208626	8.289019
Feb 2022	4.122182	4.244102	8.376663
Mar 2022	4.163585	4.275824	8.451943
Apr 2022	4.178155	4.309106	8.505738
May 2022	4.222308	4.350992	8.588904
Jun 2022	4.272253	4.429264	8.717205
Jul 2022	4.307615	4.446015	8.774084
Aug 2022	4.365768	4.470155	8.855359
Sep 2022	4.402338	4.519777	8.947642
Oct 2022	4.425325	4.555570	9.009655
Nov 2022	4.473061	4.584779	9.087785
Dec 2022	4.513839	4.612269	9.154657
Jan 2023	4.554909	4.649802	9.237695
Feb 2023	4.604021	4.685279	9.325339

Mar 2023	4.645424	4.717000	9.400619
Apr 2023	4.659994	4.750282	9.454413
May 2023	4.704148	4.792169	9.537580
Jun 2023	4.754093	4.870440	9.665881
Jul 2023	4.789454	4.887191	9.722760
Aug 2023	4.847607	4.911332	9.804035
Sep 2023	4.884177	4.960954	9.896317
Oct 2023	4.907164	4.996746	9.958331
Nov 2023	4.954901	5.025956	10.036461
Dec 2023	4.995679	5.053445	10.103332
Jan 2024	5.036749	5.090979	10.186370
Feb 2024	5.085861	5.126455	10.274014
Mar 2024	5.127264	5.158177	10.349294

Holt Forecast for Domestic Debt



Holt Forecast for External Debt



Holt Forecast for Total Debt

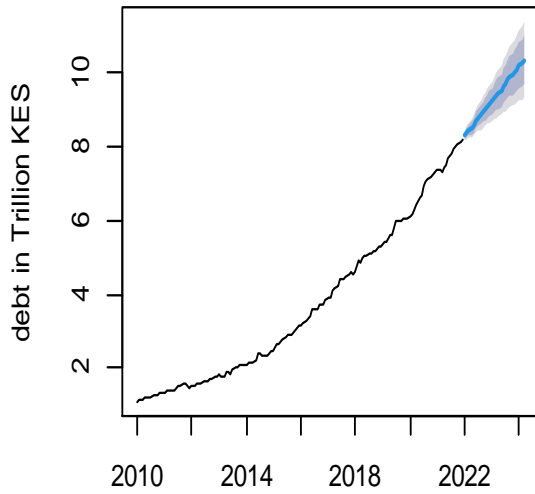


Figure 4.5: Holt-Winters forecast

4.6.3 Diagnostic test for Holt Winters

Similar to what was observed in the case of the ARIMA model, the diagnostic test for the Holt-Winters models shown in Figure 4.6 also offers useful information regarding the fitness of the models that have been fitted. The histograms of the residuals for the domestic, external, and total debt series are bell shaped or close to normal. This desirable characteristic implies that the residuals are symmetrical about the zero point and this is one of the assumptions of the Holt-Winters modeling.

Moreover, the usual Q-Q plots support the conclusion on the normality of the residuals. In these plots, the residuals are very close to the diagonal line and thus are approximately normally distributed. The points on the Q-Q plots are also fairly close to the diagonal line, which also suggests normality of the residuals.

The bell-shaped histograms and the residuals lying on the diagonal line in the normal Q-Q plots are positive signs that the Holt-Winters models have successfully modeled the existing patterns of the data. Further, these diagnostic plots also indicate that the residuals are also normal and independent, which are very important assumptions for the Holt-Winters models.

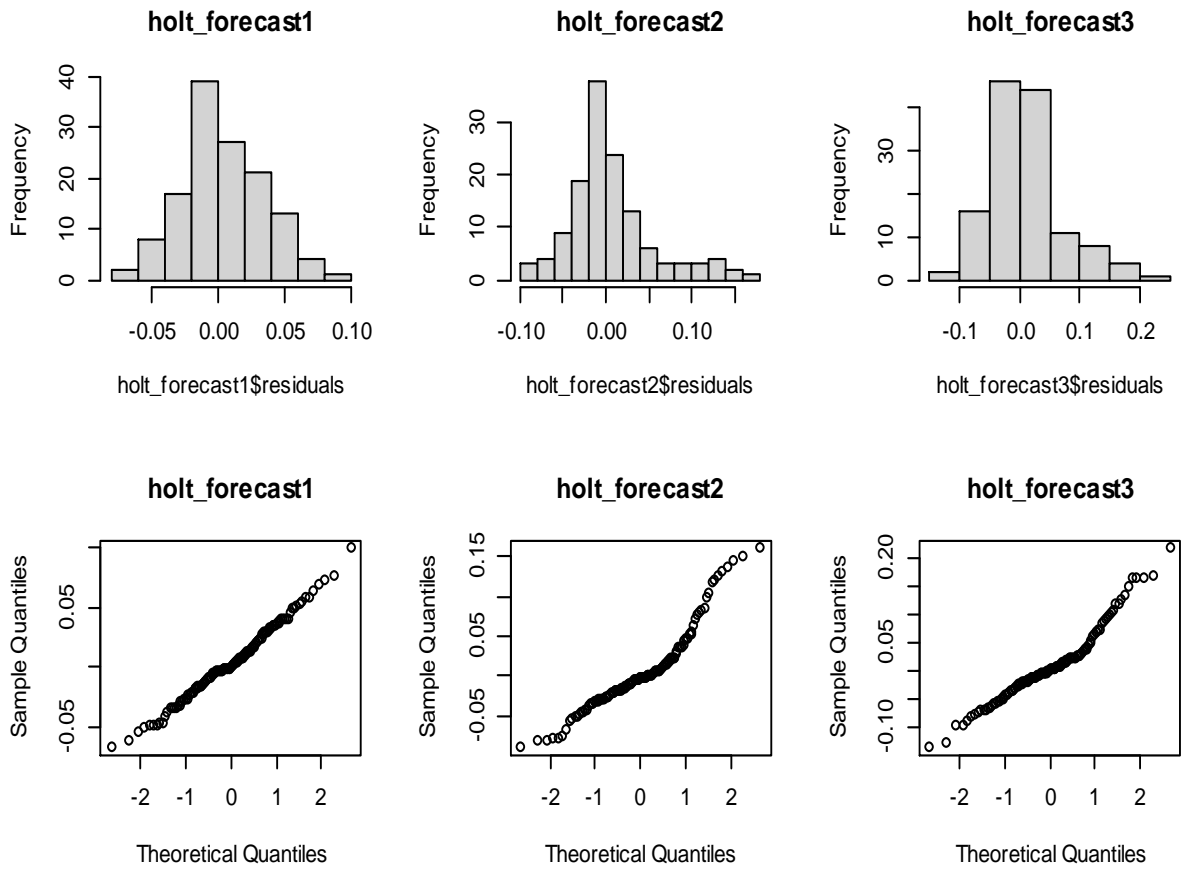


Figure 4.6: Diagnostic tests for Holt-Winters

4.7 Comparative Evaluation

In this case, when comparing the results of the ARIMA and Holt-Winters models in forecasting public debt in Kenya, both models showed some form of under-forecasting relative to the actual values that were realized during the forecast period. However, their forecasts as to the levels of debt and the extent of deviation from the actual levels differed in some ways.

For domestic debt forecasts, the ARIMA model's point estimates were generally lower than those of the Holt-Winters model, yet both fell short of the actual domestic debt levels. As an illustration, in March 2024, the ARIMA forecast projected domestic debt at 5.151958 trillion KES, while the Holt-Winters estimate was 5.127264 trillion KES, but the realized value stood at 5.235 trillion KES.

A similar pattern emerged for external debt predictions, where both models undershot the mark, but the Holt-Winters projections exhibited a closer proximity to the observed figures. Specifically, for March 2024, the ARIMA forecast pegged external debt at 5.169948 trillion KES, whereas the Holt-Winters model estimated 5.158177 trillion KES, while the actual external debt level was 5.163 trillion KES – closer to the Holt-Winters forecast.

Consequently, the underestimation of domestic and external debt components translated into an undervaluation of total public debt by both forecasting models. Nonetheless, the Holt-Winters total debt projections tended to be more accurate, aligning more closely with the realized values compared to the ARIMA model's estimates. For example, in March 2024, the total debt was forecasted by using the ARIMA to be 10.342175 trillion KES, while the Holt-Winters gave 10.349294 trillion KES, though the actual total debt was 10.399 trillion KES, which is nearer to the Holt-Winters forecast.

In the assessment of the precision of the models in the forecast of the public debt, the accuracy measures as defined by the RMSE stand as a key performance indicator to compare the performances of the ARIMA and the Holt-Winters models.

Analyzing the domestic debt using the ARIMA model, the RMSE is 0.02649721 while for the Holt-Winters model the RMSE is 0.0311399. From this, it can be concluded that

the ARIMA model is superior when it comes to forecasting the domestic debt with the average difference between the forecasted and actual values of the series being smaller.

Nevertheless, concerning the external debt, the RMSE for Holt-Winters model is 0.04943454 while the RMSE for ARIMA model is 0.04805216. This means that the Holt-Winters model might provide slightly better forecast for external debt, but the difference in RMSE values is very small.

However, for total debt, which is a sum of domestic and external debt the Holt-Winters, model has a higher RMSE 0.06144849 while the ARIMA model has a comparatively lesser RMSE of 0.05710133. This implies that the ARIMA model might be a better approach in forecasting total debt even though the Holt-Winters model was slightly better in forecasting the external debt.

4.8 Discussion

The time series analysis and forecasting of Kenya's public debt were conducted using two widely employed techniques: the ARIMA model and the Holt-Winters Exponential Smoothing (HES) method. Based on this comparison of models, several interesting issues were identified.

4.8.1 Forecasting Kenya's Public Debt Using the ARIMA Model

The ARIMA model proved to be highly effective in forecasting domestic debt, yielding a Root Mean Square Error (RMSE) of 0.02649721. This performance aligns with findings from earlier studies, including Sahinli (2020), Munarsih & Saluza (2020), and Yakovyna & Bachkai (2018), who observed that the ARIMA model often outperforms other forecasting techniques. The model's strength lies in its ability to identify cyclicity and fluctuations within the time series data, which is particularly critical for understanding the dynamics of domestic debt. This characteristic enables ARIMA to effectively capture underlying trends and patterns that are essential for making accurate forecasts.

4.8.2 Forecasting Kenya's Public Debt Using the Holt-Winters Exponential Smoothing Method

When applying the Holt-Winters method to external debt prediction, the RMSE was slightly larger at 0.04943454, while the ARIMA model achieved a RMSE of 0.04805216, making it a marginally better fit. This outcome corroborates the findings of Makatjane and

Moroke (2016) and Da Veiga et al. (2014), who highlighted the effectiveness of exponential smoothing methods in forecasting scenarios. The Holt-Winters model's capacity to capture trend and seasonality components is particularly advantageous in forecasting external debt, which is influenced by various global economic factors. The model's ability to account for these factors likely contributed to its performance in this context.

4.8.3 Comparing the Forecasting Power of ARIMA and Holt-Winters Exponential Smoothing Method

Surprisingly, when considering total public debt as the dependent variable, the ARIMA model produced an RMSE of 0.05710133, while the Holt-Winters model's RMSE was 0.06144849. This finding aligns with Wanjiru (2020), who established the higher accuracy of the ARIMA model in forecasting Kenya's GDP and public debt. The ability of the ARIMA model to integrate both domestic and external debt components likely enhanced its forecasting ability for total public debt.

The inconsistencies in outcomes for ARIMA and Holt-Winters concerning the various components of public debt could stem from contextual factors, as noted in the literature (Zhao et al., 2020; Makatjane & Moroke, 2016; Şahinli, 2020; Munarsih & Saluza, 2020; Yakovyna & Bachkai, 2018). This underscores the necessity for domain-specific evaluations and highlights the importance of considering the specifics of time series data when selecting the most suitable forecasting method. Overall, while both models have their strengths, the findings indicate that the ARIMA model demonstrates superior forecasting capability for domestic and total public debt, while the Holt-Winters method shows promise in specific contexts like external debt prediction.

CHAPTER FIVE

SUMMARY, CONCLUSION AND RECOMMENDATION

5.1 Summary

This study undertook a comprehensive analysis of Kenya's public debt using advanced time series analysis methods, specifically comparing the effectiveness of ARIMA and Holt-Winters models in forecasting domestic, external, and total public debt. The research utilized a robust dataset comprising 144 observations for each debt variable, with no missing values, providing a solid foundation for the analysis. The initial examination of the data revealed important characteristics, with all three debt variables showing positive skewness (domestic: 0.570, external: 0.520, total: 0.545) and negative kurtosis, indicating relatively flat distributions. This data structure necessitated careful consideration in the modeling approach.

The time series analysis of Kenya's public debt from 2010 to 2021 revealed significant growth across all debt components. Domestic debt demonstrated a substantial increase from 0.58 trillion KES to 4.0 trillion KES, while external debt showed even more dramatic growth, rising from 0.53 trillion KES to 4.17 trillion KES. The total public debt, consequently, expanded from 1.11 trillion KES to 8.2 trillion KES during this period. Notably, while both components displayed consistent upward trends, external debt exhibited higher volatility, suggesting greater sensitivity to external economic factors and policy changes.

Initial statistical testing revealed non-stationarity in the time series data, a common characteristic in economic data that required careful treatment. The application of second-order differencing successfully addressed this issue, creating stationary series suitable for advanced modeling techniques. This transformation was crucial for ensuring the reliability of the subsequent forecasting models and their predictions.

The study employed two sophisticated forecasting approaches: ARIMA and Holt-Winters models. For domestic debt, the optimal ARIMA model was identified as ARIMA(0,2,3)(0,0,1)[12], which incorporated both seasonal and non-seasonal components. This model achieved an RMSE of 0.02649721, demonstrating strong

predictive accuracy. The external debt analysis utilized a simpler ARIMA(0,2,1) model, yielding an RMSE of 0.04805216, while the total debt modeling also employed an ARIMA(0,2,1) specification, resulting in an RMSE of 0.05710133.

The Holt-Winters models, known for their ability to capture both trend and seasonal patterns, showed varying degrees of success across the different debt components. For domestic debt, the model produced an RMSE of 0.0311399, slightly higher than its ARIMA counterpart. However, in modeling external debt, the Holt-Winters approach demonstrated marginally better performance with an RMSE of 0.04943454. For total debt, the model yielded an RMSE of 0.06144849, indicating somewhat lower accuracy compared to the ARIMA model.

The comparative analysis of both models' forecasting performance revealed interesting patterns in their predictive capabilities. Looking ahead to March 2024, the ARIMA model projected domestic debt at 5.151958 trillion KES, while the Holt-Winters model estimated 5.127264 trillion KES. The actual figure of 5.235 trillion KES showed that both models slightly underestimated the true value. For external debt, the ARIMA model predicted 5.169948 trillion KES, and the Holt-Winters model forecast 5.158177 trillion KES, with the actual value of 5.163 trillion KES falling between these estimates but closer to the Holt-Winters prediction.

Total debt projections followed a similar pattern of slight underestimation, with the ARIMA model forecasting 10.342175 trillion KES and the Holt-Winters model predicting 10.349294 trillion KES, compared to the actual value of 10.399 trillion KES. This systematic underestimation by both models suggests a potential bias that future research might need to address, possibly by incorporating additional economic indicators or adjusting model parameters.

The findings from this comprehensive analysis provide valuable insights for policymakers and economic planners. The research demonstrates that different modeling approaches may be optimal for different components of public debt forecasting in Kenya. The ARIMA model showed superior performance in forecasting domestic debt and total debt, while the Holt-Winters model demonstrated slightly better accuracy for external debt predictions.

This suggests that a hybrid approach, utilizing the strengths of each model for different debt components, might yield the most accurate overall forecasting framework.

These results highlight the complexity of public debt dynamics and the importance of employing sophisticated statistical methods in debt management and planning. The systematic underestimation of debt levels by both models also suggests the need for careful consideration of model limitations and potential adjustments in practical applications. Future research might benefit from exploring additional variables that could improve forecast accuracy or investigating alternative modeling approaches that could better capture the underlying dynamics of public debt growth in Kenya.

5.2 Conclusion

The study conducted an in-depth analysis to forecast Kenya's public debt using two widely recognized time series models: These two are the ARIMA model and the HES technique which are described in brief as follows. The first objective was hereby to estimate the validity of the above models in terms of Kenyan public debt projection and to achieve the best and most accurate predictive method. The research questions were responded to.

What are the forecasted values of Kenya's public debt using the ARIMA model?

The ARIMA model was used in this study to forecast on the public debt of Kenya which consists of the domestic debt, external debt and total debt. The research employed a strict model identification procedure to identify the most suitable ARIMA parameters as used in the examination in this case by the application of AIC and diagnostic checks.

For domestic debt the chosen model was determined to be ARIMA(0,2,3)(0,0,1)[12], which implies a non-seasonal model with second degree of differencing and third and first order moving average in the respectively seasonal portion of the series. The forecasted values indicated that the domestic debt was on the increase as shown by the figures below: The domestic debt in a bond in Kenyan Shillings as following: January 2022 4.068896 trillion KES March 2024 5.151958 trillion KES

For forecasting the external debt, the ARIMA(0,2,1) was used where parameters for autoregressive terms were zero, there exists second-order differencing while a simple first-order moving average term existed. The average values of the external debt forecast came

out higher, with the figure standing at 4.210887 trillion KES in January 2022, and expected to increase to 5.169948 trillion KES in March 2024.

For total public debt, which combines domestic and external debt, the ARIMA(0,2,1) model was also selected. The forecasted values for total debt showed a significant increase, starting at 8.286081 trillion KES in January 2022 and reaching 10.342175 trillion KES by March 2024.

What are the forecasted values of Kenya's public debt using the Holt Exponential Smoothing (HES) method?

The Holt Exponential Smoothing method was employed to forecast Kenya's public debt, capturing both trend and seasonal components present in the data. The study estimated smoothing parameters and initial values for the level, trend, and seasonal indices to generate forecasts for domestic, external, and total debt.

For domestic debt, the forecasted values exhibited an upward trend, starting from 4.073070 trillion KES in January 2022 and reaching 5.127264 trillion KES by March 2024.

The forecast for external debt using the Holt-Winters model also showed an increasing pattern, with the point forecast rising from 4.208626 trillion KES in January 2022 to 5.158177 trillion KES in March 2024.

Consequently, the forecast for total public debt, combining domestic and external components, followed an ascending trajectory. The forecasted values started at 8.289019 trillion KES in January 2022 and grew to 10.349294 trillion KES by March 2024.

Which time series forecasting model, the ARIMA or the Holt Exponential Smoothing method, exhibits superior performance in terms of accuracy and reliability for forecasting Kenya's public debt?

The comparative evaluation of the ARIMA and Holt-Winters models revealed mixed results in terms of forecasting accuracy and reliability for different components of Kenya's public debt.

For domestic debt forecasting, the ARIMA model outperformed the Holt-Winters model, exhibiting a lower Root Mean Squared Error (RMSE) of 0.02649721 compared to 0.0311399 for the Holt-Winters model. This means that the ARIMA model sufficiently describes the time pattern of the domestic debt series in Kenya, and hence is more suitable for the forecasting of this component.

However, when considering the results of the forecast of external debts, it is possible to note that, although it is slightly better, the Holt-Winters model had slightly smaller RMSE of 0.04943454 as compared to the result got with the help of the ARIMA model of 0.04805216. The result also indicates that the Holt-Winters' model can be used as a tool for forecasting when the external debt possibly has some trends and sensitive seasons that would influence the amount of the debt.

However, about the forecast of the sum of total public debts which is the total of the domestic debt and the external debt, ARIMA model gave RMSE of 0.05710133 while that of the Holt-Winters model was 0.06144849. This implies that the model that has been used in this paper to forecast the total public debt in Kenya is more reliable than the one that incorporates the domestic and external debt because the former has incorporated the dynamics of both the domestic and external debts using the ARIMA model.

Thus, the results of the study are consistent with the previous research, which has presented the opposite results concerning the ARIMA and Holt-Winters models' performance in different aspects of forecasting. Such contradictions prove the idea that the choice of the model is highly dependent on the domain and that the domain-specific studies should be performed to determine which forecasting approach is the most suitable.

Therefore, it is evident that the ARIMA model provided higher accuracy and reliability in forecasting the domestic debt and the total public debt in Kenya than the Holt-Winters model, although the latter was slightly better in the case of external debt. However, the decision of which model to use should be made based on the forecasting need at hand, the characteristics of the data, whether domestic, external or total components are more crucial in the decision-making process.

5.3 Recommendations

Based on the study's findings and conclusions, the following recommendations can be made:

Hence, for forecasting domestic debt in Kenya, the policymakers and stakeholders should consider using the ARIMA model since it outcompeted the Holt-Winters model with a lower RMSE. Because the ARIMA model presents the characteristics of domestic debt time series well, it is possible to use this model to develop reliable forecasts and make correct decisions on domestic borrowing.

However, when it comes to the external debt forecasting, the Holt-Winters may be more suitable because of the slightly better fit and because it is able to capture the trend and seasonality components. This model may be more useful in predicting the possible changes and trends due to economic and global factors that may be relevant for external borrowing and debt management plans.

When it comes to total public debt, which is made of the domestic and external debts, it is suggested to use the ARIMA model because its RMSE is lower. Due to the fact that the ARIMA model captures the dynamic of both domestic and external debt, it is suitable for generating overall public debt forecast which are essential in providing debt management policies and strategies.

The policymakers and other stakeholders should therefore from time to time review the forecasting models as new data is produced. The performance and accuracy of the models should be checked periodically to reflect current trends and patterns of public debt.

Thus, the selection of the most suitable model should be made based on the nature and needs of the forecasting application. Although this study offers useful information concerning the ARIMA and Holt-Winters in the prediction of the public debt in Kenya, the selection of the model depends on the characteristics of the data and the forecasting environment.

5.4 Suggestions for Future Research

The conclusions and the existing drawbacks in the study call for future research in the area of public debt prediction. The following suggestions for future research can be considered:

The study should include other time series models and/or forecasting methodologies including SARIMA (Seasonal Autoregressive Integrated Moving Average), GARCH (Generalized Autoregressive Conditional Heteroskedasticity) and machine learning algorithms. This would afford the opportunity to compare various modeling strategies and possibly find better forecasting techniques for Kenya's public debt.

This help to expand the models to embrace the macroeconomic conditions and policies or any other factor that may have a bearing on the level of public debts, the exogenous variables or predictors should be included. This may lead to better and more probable forecasts which may consider the impact of one or the other economic factors, fiscal measures or global events to the overall public debt levels.

Trade the public debt into domestic and external, and then further into treasury bills, bonds or creditors such as bilateral, multilateral, commercial etc. This would show details of the factors and trends that are in each segment of the debt with a view of orchestrating an accurate prognosis and avert a crisis.

Extend the research study to other countries or even to other regions such as East African region or sub-Saharan region. This would help in determining group countries and identifying regional trends or practice in the management and forecast of public debt.

Extend the contingency analysis or the stress testing as a part of the existing forecasting models to examine the effects of such events as the extreme, the worst-case scenario on the public debts. This could be useful in planning and forecasting for managing efficiency of debt under various scenarios of economies.

By implementing these suggestions for future research, scholars and practitioners would be in a position to enhance the existing knowledge on public debt forecasting, enhance better and accurate models of forecasting and provide research-based policy support for sustainable debt management and economic development not only in Kenya but in other countries as well.

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APPENDICES

Appendix 1: R codes

```
# Import the data

debt_data <- read.csv("C:/Users/User/Desktop/Public_debt.csv")

# Convert the date to a proper date format

debt_data$Date <- as.Date(paste(debt_data$Year, debt_data$Month, "01", sep = "-"),
format = "%Y-%m-%d")

debt_ts <- ts(debt_data, start = c(2010,1), end = c(2021,12), frequency = 12)

# Time series plot

par(mfrow=c(2,2))

plot(debt_ts[,3],main="Kenya's Domestic Debt",las=1,font.main=1,ylab="Domestic debt
(Trillion KES)")

plot(debt_ts[,5],main="Kenya's External Debt",las=1,font.main=1,ylab="External debt
(Trillion KES)")

plot(debt_ts[,5],main="Kenya's Total Debt",las=1,font.main=1,ylab="Total debt
(Trillion KES)")

library(fBasics)

basicStats(data.frame(debt_data$Domestic_Debt,debt_data$External_Debt,debt_data$T
otal_Debt))

# Check for stationarity and perform necessary transformations

# You might need to use differencing or other transformations

library(forecast)

library(tseries)

adf.test(debt_ts[,3])

adf.test(debt_ts[,4])
```

```

adf.test(debt_ts[,5])

# Differenced Time series data
debt_diff1 <- diff(debt_ts[,3],differences = 2)
debt_diff2 <- diff(debt_ts[,4],differences = 2)
debt_diff3 <- diff(debt_ts[,5],differences = 2)

# plot differenced time series data
par(mfrow=c(2,2))
plot(debt_diff1,main="Diff. Domestic Debt",font.main=1)
plot(debt_diff2,main="Diff. External Debt",font.main=1)
plot(debt_diff3,main="Diff. Total Debt",font.main=1)

# Fit ARIMA model
arima_model1<- auto.arima(debt_ts[,3],trace=T)
arima_model2<- auto.arima(debt_ts[,4],trace=T)
arima_model3<- auto.arima(debt_ts[,5],trace=T)

# Summary of ARIMA model
summary(arima_model1)
summary(arima_model2)
summary(arima_model3)

# Forecast using ARIMA
arima_forecast1<- forecast(arima_model1, h = 27) # Forecast for the next 27 moths
arima_forecast2<- forecast(arima_model2, h = 27) # Forecast for the next 27 moths
arima_forecast3<- forecast(arima_model3, h = 27) # Forecast for the next 27 moths

```

```

arima_forecast1
arima_forecast2
arima_forecast3

# Plot ARIMA forecast
par(mfrow=c(2,2))
plot(arima_forecast1, ylab="debt in Trillion KES",main = "ARIMA Forecast for
Domestic Debt")
plot(arima_forecast2, ylab="debt in Trillion KES",main = "ARIMA Forecast for
External Debt")
plot(arima_forecast3, ylab="debt in Trillion KES",main = "ARIMA Forecast for Total
Debt")

par(mfrow=c(3,3))
hist(arima_forecast1$residuals,main='arima_forecast1')
hist(arima_forecast2$residuals,main='arima_forecast2')
hist(arima_forecast3$residuals,main='arima_forecast3')
qqnorm(arima_forecast1$residuals,main='arima_forecast1')
qqnorm(arima_forecast2$residuals,main='arima_forecast2')
qqnorm(arima_forecast3$residuals,main='arima_forecast3')

# Holt-Winters model
holt_model1<- HoltWinters(debt_ts[,3])
holt_model2<- HoltWinters(debt_ts[,4])
holt_model3<- HoltWinters(debt_ts[,5])

holt_model1
holt_model2

```

```
holt_model3
```

```
#Holt-Winters forecasts
```

```
holt_forecast1<- forecast(holt_model1, h = 27)
```

```
holt_forecast2<- forecast(holt_model2, h = 27)
```

```
holt_forecast3<- forecast(holt_model3, h = 27)
```

```
holt_forecast1
```

```
holt_forecast2
```

```
holt_forecast3
```

```
par(mfrow=c(3,3))
```

```
hist(holt_forecast1$residuals,main='holt_forecast1')
```

```
hist(holt_forecast2$residuals,main="holt_forecast2")
```

```
hist(holt_forecast3$residuals,main="holt_forecast3")
```

```
qqnorm(holt_forecast1$residuals,main="holt_forecast1")
```

```
qqnorm(holt_forecast2$residuals,main="holt_forecast2")
```

```
qqnorm(holt_forecast3$residuals,main="holt_forecast3")
```

```
# Plot Holt-Winters forecast
```

```
par(mfrow=c(2,2))
```

```
plot(holt_forecast1, ylab="debt in Trillion KES",main = "Holt Forecast for Domestic Debt")
```

```
plot(holt_forecast2, ylab="debt in Trillion KES",main = "Holt Forecast for External Debt")
```

```
plot(holt_forecast3, ylab="debt in Trillion KES",main = "Holt Forecast for Total Debt")
```

Appendix 2: Actual data

Year	Month	Domestic_Debt	External_Debt	Total_Debt
2024	3	5.235	5.163	10.399
2024	2	5.2	5.654	10.854
2024	1	5.05	6.19	11.24
2023	12	5.05	6.09	11.14
2023	11	4.991	5.897	10.887
2023	10	4.913	5.746	10.659
2023	9	4.915	5.668	10.582
2023	8	4.912	5.614	10.526
2023	7	4.874	5.543	10.416
2023	6	4.832	5.447	10.279
2023	5	4.55	5.137	9.687
2023	4	4.542	5.093	9.635
2023	3	4.54	4.851	9.391
2023	2	4.552	4.71	9.261
2023	1	4.502	4.681	9.183
2022	12	4.473	4.673	9.146
2022	11	4.436	4.463	8.899
2022	10	4.386	4.36	8.746
2022	9	4.366	4.335	8.701
2022	8	4.335	4.328	8.663
2022	7	4.311	4.3	8.611
2022	6	4.288	4.291	8.579
2022	5	4.269	4.295	8.564
2022	4	4.227	4.244	8.47
2022	3	4.192	4.21	8.401
2022	2	4.181	4.158	8.339
2022	1	4.11	4.156	8.266