

**FORECASTING DOMESTIC TAX REVENUES IN KENYA USING SARIMA
AND HOLT-WINTERS METHODS**

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DECLARATION

This project is my original work and has not been presented elsewhere for a degree or any other award.

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DEDICATION

I dedicate this research work to my beloved parents and siblings, whose unwavering support and encouragement throughout my academic journey have laid the foundation for this significant achievement. Your prayers and best wishes have been a steadfast pillar, especially during my most challenging moments.

To my classmates and friends: thank you for your invaluable support and encouragement during the research analysis and the demanding publication period. Your camaraderie made the journey both manageable and memorable.

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LIST OF ABBREVIATIONS/ACRONYMS

ADF	Augmented Dickey-Fuller
AIC	Akaike Information Criteria
ARIMA	Autoregressive integrated moving average
BIC	Bayes-Schwarz Information Criteria
CIT	Corporate Income Tax
DM-Test	Diebold-Mariano test
DRM	Domestic Resource Mobilization
DTD	Domestic Taxes Department
FY	Financial Year
GDP	Gross Domestic Product
HW	Holt-Winters
IEA	Institute of Economic Affairs
KIPPRA	Kenya Institute for Public Policy Research and Analysis
KRA	Kenya Revenue Authority
MAE	Mean Absolute Error
MAPE	Mean Absolute Percentage Error
MASE	Mean Absolute Scaled Error
MCS	Model Confidence Set
MSE	Mean Square Error
PACF	Partial Autocorrelation Function test
PAYE	Pay As You Earn
PP	Phillips-Perron test
RMSE	Root Mean Square Error
SARIMA	Seasonal autoregressive integrated moving average
SDGs	Sustainable Development Goals
VAT	Value Added Tax

LIST OF SYMBOLS

α_0	time series model constant
ε_t	random errors at time-period t
y'_t	differenced series
p	order of autocorrelation
d	order of integration
q	order of moving averages
a_t	nonstationary time series
s	number of periods per season
$\varphi(B)$	Non-seasonal autoregressive of order p
$\theta(B)$	Non-seasonal moving average of order q
$\Phi_p(B^s)$	Seasonal autoregressive components with orders P
$\theta_Q(B^s)$	Seasonal moving average components of Q
∇^d	Ordinary/non-seasonal difference components
∇_s^D	seasonal difference components
B	backshift operator.
Z_t	product of seasonal differencing D
L_t	Holt Winter Series level
b_t	trend component
S_t	seasonal component
F_{t+m}	forecast for the m periods ahead
α	Alpha smoothing parameters
β	Beta smoothing parameter
γ	Gamma smoothing parameter
X_t	Forecast for financial year t

ABSTRACT

Many countries rely on tax revenues to finance their expenditures; thus, forecasting revenue is important in fiscal planning, policy formulation, and fiscal decision-making. There have been ongoing discussions about Kenya's budget-making process and whether its revenue estimates are realistic. Over the last 13 fiscal years, revenue collections in Kenya have increased tremendously by 263% (from KES 0.707 trillion in 2011/2012 to KES 2.571 trillion in 2024/25). This growth has been attributed to numerous government efforts, including expanding the tax base, changing tax rates, increasing voluntary compliance, and enhancing revenue mobilization. Despite this remarkable revenue growth, the set revenue targets have seldom been met. Both underestimation and overestimation of tax revenue have led to economic instability. For this reason, it is prudent for the country to explore scientific forecasting methods, such as time series analysis, since tax revenue is collected over time. The failure to meet targets by revenue collectors is often due to the setting of higher targets, driven by the ambition to reach a certain percentage of Gross Domestic Product (GDP), as well as inefficiencies in tax administration and inaccurate forecasts. Failure to meet the revenue targets has often led to unmet expenditure commitments, which have led to increased domestic and foreign borrowing in Kenya. The primary aim is to fit a suitable model that can be used in forecasting domestic revenues in Kenya using the SARIMA and HW time series methods, compare their performances, and use them to create a 1-year forecast. The forecasting process involved model identification, model estimation, adequacy testing, and modelling. Secondary data on the domestic taxes collected in Kenya between July 2014 to December 2020 across the various tax heads was used. Based on its minimal AIC=1358.68, BIC=1363.03, and the least forecasting errors (MAPE=7.67, MASE=0.38, and MAE=4,998.15), the SARIMA (0,1,1)(0,1,0)_[12] model was preferred compared to the Additive and Multiplicative HW methods. The predictive abilities of the three models were measured using the Diebold-Mariano test and were found to be significantly different. The Model Confidence Set procedure reaffirmed this by eliminating the Additive and Multiplicative HW models and retaining the SARIMA model as the most suitable, hence recommended for forecasting domestic tax revenues. In most months, SARIMA provided more conservative forecasts that were generally closer to actual figures, but sometimes underestimated revenue. Incorporating macroeconomic variables and advanced machine learning techniques could increase the accuracy levels and the model's reliability, hence can be explored in future studies.

CHAPTER ONE

INTRODUCTION

1.1 Background of the Study

Institutions and governments worldwide levy taxes on their subjects to collect revenues that are used to finance their expenditures. According to Harelimana (2018), taxation is a vital tool towards financing capital-intensive government projects, healthcare, education, and other important programs that are budgeted each year. This has therefore necessitated the need for governments to collect more revenues through taxation as well as sealing loopholes to ensure each individual or organization gives a fair share to the economy.

According to Kaplan and Poulson (2018), increased taxation in economies has resulted in a rise in inflation levels for both goods and services. It is also a determining factor for investors, where many of them look for opportunities in areas with lower tax regimes to maximize profits (Fisher, 2014). This, therefore, calls for balanced taxation to attract investors to increase employment opportunities for residents and also grow the Gross Domestic Product.

Domestic taxes, which may be direct or indirect, are levied on income generated within a country. In Kenya, the primary domestic taxes include Corporation Tax, Pay As You Earn (PAYE), Withholding Tax, Value Added Tax (VAT), Rental Income Tax, Excise Duty, Advance Tax, Personal Income Tax (PIT), Turnover Tax, and Capital Gains Tax (Moyi & Ronge, 2006).

An article by Amadala (2021) highlighted to the Kenya National Treasury the necessity of a stronger revenue forecasting capacity. Ofori *et al.* (2020) emphasizes that accurate tax revenue predictions are crucial for effective economic planning. Further, IEA Kenya affirms that this will prevent over-ambitious budget plans that drive the country away from constant borrowing. The revenue forecasts rose from 5.5% to over 19% between 2014 to 2020. This increased growth in projection was questioned by the IEA, highlighting the need for improved predictability of public funds for better budgeting.

In Kenya, revenue forecasts are released by the Ministry of Finance, in partnership with other stakeholders such as KIPPRA. The National Treasury then develops government budget estimates based on these forecasts, which are submitted to Parliament for approval.

These revenue projections form the basis for annual revenue targets assigned to the Kenya Revenue Authority (KRA) for implementation.

The Kenyan government has implemented numerous tax reforms aimed at increasing tax revenues to fund the budget without excessive borrowing. This effort aligns with the broader goal of internal revenue mobilization to meet public expenditure needs (Signé, 2016). However, challenges such as corruption, fraud, and tax evasion remain significant obstacles to revenue collection. Lilian (2015) reported that tax evasion schemes by multinational companies result in an annual loss of Ksh. 639 billion.

The seasonal nature of certain tax categories, such as Value Added Tax (VAT) and import duties, presents challenges for tax revenue forecasting (Samuel & Kibua, 2019). Nonetheless, the use of appropriate revenue forecasting models is essential for effective fiscal planning and projection, ultimately leading to reliable tax revenue forecasts.

Common methods for tax revenue forecasting include judgmental approaches, constant trend growth models, and the application of static and dynamic models.

1.1.1 Judgmental methods

This model relies on the intuitive judgment or opinion of the analyst based on knowledge of the drivers of revenue and the prior actual cash flow information (Boonzaaier, 2012). This method could lead to an underestimation of projected revenue in instances where organizations pay their tax obligations in advance. Conversely, if taxpayers are expected to pay a certain tax obligation but fail to do so after the due date, this leads to an overestimated projection. This highlights the need for a more scientific forecasting method, such as time series analysis.

1.1.2 The constant trend growth Model

In this method, the present year's projections are based on the presumption of consistent growth throughout the year.

This is derived from the equation;

$$X_t = \frac{XT_t}{XT_{t-1}} * X_{t-1}, \tag{1}$$

where;

Considering the growth rate $f_t = \frac{XT_t}{XT_{t-1}} - 1 \therefore \frac{XT_t}{XT_{t-1}} = 1 + f_t$

Substituting the above in equation (1) $\Rightarrow X_t = (1 + f_t) * X_{t-1}$ (2)

X_t is the forecast of period t , f_t is the growth rate of XT for the period between $t - 1$ and t , XT_t is the cumulative actual figure for period t , XT_{t-1} is the cumulative actual figure for the previous period $t - 1$, and X_{t-1} is the actual figure realized for the period $t - 1$.

However, this method assumes a similar growth rate in the whole financial year, which is often unrealistic given that there are many factors that impact an economy. This highlights the need for a more refined forecasting model.

1.1.3 Static and dynamic models

Static models are typically based on GDP. These models require compiling a series of tax revenues and the corresponding tax bases for the taxes. Forecasting in this context primarily relies on predetermined frameworks for different types of taxes.

Dynamic models, on the other hand, are generally more comprehensive because they consider how tax bases respond when discretionary changes are introduced into the tax system (Jenkins & Shukla, 2000). They account for the behavioral effects within economic sectors when new tax regimes are implemented or current tax laws are amended, recognizing that tax bases are not fixed when projecting revenues. As a result, complex econometric models are necessary, but the extensive information required often makes them impractical for many countries.

Due to limited access to advanced computing skills and information, static models remain prevalent. Although dynamic models offer many potential advantages, they sometimes fail to fully capture changes in tax revenue, which can lead to less accurate forecasts.

1.2 Problem Statement

The ever-growing size of Kenya's budget has steered discussions regarding the capability of the country to fund its budget. This is as a result of the increase in borrowing in the country and continuous dependence on foreign and domestic debt to fund its expenditures (Jenkins & Shukla, 2000). For instance, the national budget expanded from Ksh. 2.20

trillion in FY 2015/2016 to Ksh. 3.02 trillion in FY 2019/2020 and Ksh. 4.29 trillion in the FY 2025/2026 (National Treasury and Planning, 2025). This represents an approximate growth of 90.9% over the last 10 years. ICPAK (2016) highlights the immense role played by revenue forecasting and accurate tax analysis in budget planning and efficient economic decision-making.

According to the KRA end year reported revenues between 2012 and 2025, the tax collected more than doubled. Specifically, from Kshs. 0.707 trillion in the budget year 2011/12 to Kshs. 1.607 trillion and Kshs. 2.571 trillion in the budget period 2019/20 and 2024/25, respectively. This constituted a 263% growth. This growth was attributed to initiatives such as taxpayer-base expansion, voluntary compliance programs, enhanced revenue mobilization efforts, and changes in tax rates. Despite this significant increase, the revenue targets have rarely been met. The failure to achieve these targets is mainly due to overly ambitious goal-setting driven by the desire to meet specific GDP targets, inefficiencies in tax administration, and inaccurate forecasting (Chimilila, 2017). Missing these targets places pressure on the government and tax authorities, as it hampers the fulfillment of national expenditure commitments, leading to increased domestic and foreign borrowing. Employing accurate forecasting methods can help set more realistic revenue collection targets. Kyobe and Danninger (2005) noted that about 85% of the 34 developing countries surveyed relied on subjective judgments and simple extrapolation methods for their revenue forecasts.

However, little empirical research has compared advanced time series models such as SARIMA and Holt-Winters in the Kenyan domestic tax revenue context, leaving a gap in evidence-based model selection for fiscal planning.

1.3 Justification of the Study

Revenue forecasting is an essential tool for enhancing the effectiveness of fiscal policy implementation. There has been an ongoing discussion on Kenya's budget-making process and whether its revenue estimates are realistic. This has been occasioned by the inability of KRA to meet the set treasury targets over several years. Besides the unmet set targets, the Kenyan government has continually increased its expenditures, hence exceeding the revenue collections in the country. For instance, in the 2015/16 financial year, government expenditure rose significantly to Ksh. 787.5 billion from Ksh. 676.4

billion in the financial year 2011/12. Between these two periods, the Kenya Gross Domestic Product (GDP) averaged 19.9%, with the mean expenditure standing at approximately 30.1% (KIPPRA, 2016), resulting in a fiscal gap of 10.2%. These findings will therefore assist policymakers in assessing the reliability of their revenue forecasts. Considering that Kenya is highly reliant on the exchequer revenue, accurate forecasting is essential in narrowing the gap between the budgeted revenue and the actual spending. This study evaluates the efficiency of existing revenue forecasting methods and explores alternative approaches better suited to Kenya's economic dynamics.

1.4 Objectives

1.4.1 General Objective

To forecast Domestic tax revenues in Kenya using the SARIMA and Holt-Winters time series methods.

1.4.2 Specific Objectives

1. To examine the main contributors of the Domestic Taxes Department (DTD) annual tax revenues in Kenya.
2. To forecast Kenya's Domestic tax revenues using SARIMA and Holt-Winters time series models.
3. To determine the best model for forecasting the Domestic tax revenues in Kenya.

1.5 Hypotheses

1. There is no significant difference in the proportion of revenue contributed by various domestic taxes in Kenya.
2. There is no significant difference in Domestic tax revenues forecasted using the SARIMA and Holt-Winters models
3. There is no difference in the prediction accuracy of Domestic tax revenues between the SARIMA and Holt-Winters models

1.6 Scope of the Study

Secondary data from the Kenya Revenue Authority (KRA), monthly revenue collection figures from July 2014 to December 2020, were used. The data included collections from major tax heads in domestic taxes such as Pay As You Earn, Value Added Tax, Corporate and Personal Income Taxes, and Excise tax, among others.

CHAPTER TWO

LITERATURE REVIEW

2.1. Background Information

The forecasting of future occurrences is mainly dependent on the predictive power of the methods utilized. Additionally, the analyst's knowledge regarding the study variables and the data is crucial in determining the forecast accuracy of models.

Jenkins *et al.* (2000) highlights that revenue forecasting methods are broadly categorized into quantitative and qualitative methods. Qualitative methods, also known as judgmental forecasts, such as consensus forecasting, are susceptible to bias due to human judgment. These methods are commonly used when data is unavailable or unrepresentative. Although consensus forecasting can reduce forecast bias, it is time-consuming, often causing delays between forecast preparation and application.

Quantitative methods have become more prevalent due to improved data collection and technological advancements. According to Makridakis *et al.* (2008), these methods assume that historical patterns will continue; however, unexpected changes may occur within longer periods. This makes the short-term forecast more accurate and reliable.

Time series methods operate under the assumption that historical data captures the independent variables influencing the outcome. As new data becomes available, the model is updated to improve future forecasts. However, if the historical data is inaccurate, future predictions are likely to be misleading. At the Kenya Revenue Authority (KRA), errors in historical data may arise during data capture. However, automation has helped minimize such errors since revenue data is generated from the set systems. According to Pindyck and Rubinfeld (1998), time series related models produce efficient estimates but also have limitations in explaining the underlying factors driving the variables of interest.

2.2. Holt-Winters Model Review

Time series modeling has been tried in the finance, transportation, tourism, and sales sectors Koirala (2013). The HW method, which is also referred to as Triple Exponential Smoothing, is one of the popular forecasting methods that was developed by Peter Winters (Habibur *et al.*, 2016). It is known to be used for datasets that have trends and seasonal

patterns. It is also resilient to the influence of outliers and random noise in datasets (Munarsih & Saluza, 2020).

Holt-Winters and ARIMA methods have been applied by scholars recently. Many authors have supported these time series models to forecast revenues. Analysis done to the Romanian local budget by Pelinescu *et al.*(2010) resulted in improved management and decision-making for the local income and expenditures. The local authority had difficulties predicting its future revenues to assist in the budget-making process. The research used the HW method to predict the aggregate revenue figures collected by the Romanian authority and found it effective, hence recommending it to forecast its annual budget.

Suwanvijit (2013) used the historical data from January 2002 to December 2011 to estimate the rate of tourists' arrival in Malaysia, Thailand, and Indonesia using the HW model. This model used the MAPE statistic to investigate the error levels in the model and employed the resultant model in the estimation of the tourism growth rates for the next 5 years. The arrival rate experienced an approximately 17% average annual increase, which was a valuable contribution to the tourism sector and forecasting.

Rahman *et al.* (2016) used monthly revenue data from the Bangladesh Bridge Authority between July 1998 to July 2016 to forecast monthly revenue using the best additive and multiplicative HW model. The study found the Additive HW method to be more accurate and used it to predict monthly revenue for January 2021.

A comparative review of the HW method and Fuzzy Time was used by Nur Fatihah Fauzi *et al.* (2020) in the prediction of the arrival rate of visitors in Langkawi from January 2015 to December 2019. Holt-Winters model achieved a low Mean Squared Error ($7.15 * 10^8$) compared to ($2.63 * 10^9$) by Fuzzy Time Series. It was therefore selected to estimate the visitors' arrival rate for 2020 and 2021 and produced more reliable results compared to its counterpart.

Ayakeme *et al.* (2021) compared how well the ARIMA and additive/multiplicative Holt-Winters models effectively predicted the domestic revenues generated by the Bayelsa government from 2012 to 2018. ARIMA (0,1,1) resulted to performed better based on the BIC and AIC test values applied to the two models. Using the ARIMA model, they projected the domestic revenues for the state for the years 2019 to 2021 and compared

their accuracy to that of the Holt-Winters methods through MAE and MSE metrics. Similarly, ARIMA yielded better forecasts, hence recommended for future revenue forecasting.

Atoyebi *et al.* (2023) analyzed Nigeria's currency in circulation (CIC) using both additive and multiplicative Holt-Winters exponential smoothing methods with data from 1960 to 2022. They tested various smoothing parameters and found that level (0.4), seasonal (0.3), and trend (0.1) were the perfect combination that yielded a good forecast. In this study, the multiplicative method had lower error metric values, hence more accurate than the additive one. The study concluded that CIC in Nigeria is on a steady rise, driven by factors such as population growth, inflation, and a predominantly cash-based economy.

Tasi'u *et al.*, (2024) conducted a cross-comparison of the SARIMA and HW techniques to forecast Nigeria's tax revenues using a time series dataset of more than thirty years. The SARIMA (3,2,1)(0,1,1)₄ model was identified to have the lowest MAE and RMSE values (0.082 and 0.165) and outperformed the HW technique. It highlighted the shortcomings of each of the two techniques, pointing out SARIMA as a reliable tool in improving fiscal management and economic planning in Nigeria.

2.3. SARIMA Model Review

Several studies across different sectors have demonstrated a strong forecasting capability of SARIMA models.

Employing the SARIMA, HW, and ARIMA techniques, Saayman and Saayman (2008), predicted the monthly tourist arrivals in South Africa. The study revealed that SARIMA resulted in the best model compared to the rest. The study highlighted the need for considering seasonality when forecasting tourism data.

To assist the Bangladesh Water Authorities, Mahsin *et al.* (2012) collected the rainfall data from 1981 to 2010 and used it to model the expected rainfall in the country for the next 24 months. The study was meant to provide insights into the rainfall expected in the city, hence estimating the water demand levels for the residents. The AIC and the RMSE statistics were used to gauge the best rainfall predictor. The SARIMA (0,0,1)(0,1,1)₁₂ emerged as the best predictor of rainfall patterns for the following 24 months in the city.

Otu *et al.*(2014) predicted Nigeria's monthly rate of inflation using the SARIMA model. The study involved 120 observations collected from various months between 2003 to 2013. The study results were used in the estimation of inflation rates for January to March 2014. SARIMA (1,1,1)(0,0,1)₁₂ was chosen as an accurate estimating model of Nigeria's inflation rate, based on the AIC criterion. Similarly, the model was used in the estimation inflation index in Kenya quarterly. SARIMA (0,1,0)(0,0,1)₄ was selected as the most appropriate model in estimation levels of inflation in Kenya based on the performance of MAE and RMSE (Susan *et al.*,2015). This study also suggested policies that could be applied in Kenya to ensure a single-digit inflation rate is achieved.

In the cross comparison of the effectiveness of the SARIMA and structural time series techniques, Ergüven *et al.* (2015) conducted several tests on various datasets using the two techniques and highlighted the high accuracy levels witnessed in short-period estimates produced by SARIMA. In another study, short-period estimates of the departure rates of tourist visiting Taiwan was computed by Chang and Liao (2010) using SARIMA. The study estimated the monthly departure rates of tourists to different destinations in Taiwan successfully.

Makananisa (2015) reviewed the SARIMA and HW techniques in the prediction of key revenues together with the total collected taxes in South Africa using twelve periods data from 1995 to 2010. The accuracy levels of the two models were evaluated. The study discovered that the two methods were suitable and had different capabilities in the prediction of different taxes. For instance, Personal Income Tax and Value Added Tax figures were accurately estimated by SARIMA, while the HW method was superior in forecasting Corporate Income and Total Tax Revenue. The research supports the use of the two methods, especially where data has minimal economic disruptions.

Samuel and Kibua (2019) fitted a SARIMA model to forecast Kenya's tax revenues and identified SARIMA (2,0,0)(2,0,0)₁₂ to be the best predictor of Kenya's tax revenue. The predictability ability was checked using RMSE and MAE to confirm its suitability. The study was aimed at supporting long-term fiscal planning.

Kelkar *et al.* (2021) investigated the revenue generated by Southwest Airlines and used this quarterly data in the estimation of the expected revenues for the Airline in the year

2020. In this endeavor, the SARIMA(0,1,0)(0,1,1) model was the most preferred, considering its low AIC value. This technique was used in the estimation of the expected revenues even in the years when the Airline witnessed lower revenues in the COVID-19 period. Further, the study conducted the Airline's risk analysis and offered valuable insights into its ability to withstand economic challenges during this period.

CHAPTER THREE

MATERIALS AND METHODS

3.1 Source of data

The study utilized secondary data from the Kenya Revenue Authority (KRA) covering monthly collections of domestic Value Added Tax, Pay As You Earn, Corporation Tax, and total Domestic Tax revenue. The dataset spans 78 months, from July 2014 to December 2020. According to Garrett and Leatherman (2000), a minimum of 50 data points is adequate for time series modeling, so the sample size in this study exceeds the recommended threshold for applying SARIMA and Holt-Winters models. Data analysis and visualization were conducted using R and Python, both widely accepted tools in Statistics and Economics for robust time series analysis.

3.2 Design

The data analysis process involved descriptive statistics, stationarity testing, model fitting, and forecasting. Descriptive analysis provided summaries and revealed the inherent characteristics of the data. The research uses the quantitative forecasting approach, which utilizes historical data and statistical models to make predictions. The analysis process involved the following steps, as emphasized by Larmore (2016) in the creation of time series models:

- i. Plotting of the data trend to check seasonality in the data.
- ii. Examining whether the data had stationarity problems. This was confirmed by the PP and ADF tests.
- iii. Initial Model estimation using the SARIMA and HW methods. This involved model identification, analysis, and testing.
- iv. Estimation of model parameters.
- v. Diagnostic checking and model evaluation
- vi. Best model selection.
- vii. Forecasting: This entails estimating future outcomes based on present and past data. This was done after the models passed the diagnostic tests. Performance metrics were used to evaluate performance and model accuracy. Consider that:

Y_t and \hat{Y}_t are the actual and forecasted observation times t ,

$e_t = Y_t - \hat{Y}_t$ is the forecast error at time t

Further, Forecast error $e_{t+h} = Y_{t+h} - \hat{Y}_{t+h}; h - i \leq 0 : h - i > 0 \therefore \varepsilon_{t+h-i} = 0$ (3)

The Box-Jenkins Procedure summarizes the forecasting conceptual framework to be adopted by researchers. This process involves four major steps that include: Model Identification, Model Estimation, Adequacy/Diagnostic checking, and Data forecasting (Box *et al.*, 2015; Jenkins *et al.*, 2000). The steps are done repeatedly until the model adequacy is achieved.

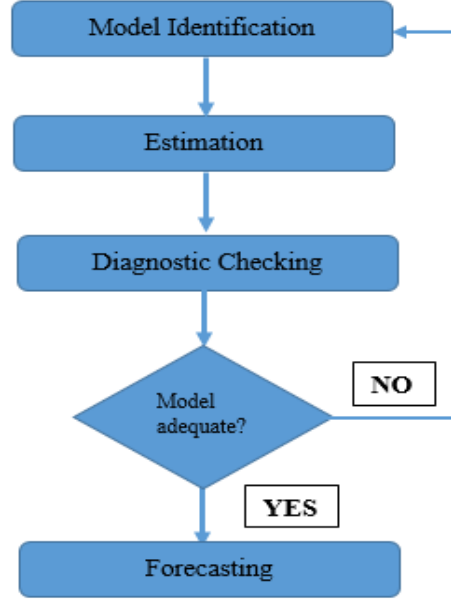


Figure 3.1: Box-Jenkins Model Principles

3.3 Accuracy Measurement

Since forecasting is the primary objective of time series modeling, the predictive accuracy of forecasts is assessed to see which models have the fewest errors (Makridakis *et al.*, 2008). To gauge the model's accuracy, RMSE, MAE, MAPE, and MASE statistics were computed. They can be expressed as:

$$\text{Mean Absolute Error (MAE)} = \frac{\sum_{i=1}^n |\varepsilon_i|}{n}$$

$$\text{Mean Absolute Scaled Error (MASE)} = \text{mean}(|a_i|),$$

$$\text{where } (a_i) = \frac{e_i}{\frac{1}{T-1} \sum_{i=2}^T |y_t - y_{t-1}|} ; t = 1, 2 \dots T \text{ sample periods}$$

$$\text{Root mean squared error (RMSE)} = \sqrt{\frac{\sum_{i=1}^n \varepsilon_i^2}{n}}$$

$$\text{Mean absolute percentage error(MAPE)} = \frac{\sum_{i=1}^n |PE_i|}{n}$$

$$\text{where Percentage error (PE)} = \frac{Y_t - F_t}{Y_t} * 100$$

Y_t is the Actual value, F_t is the Estimated value

and $e_i = Y_i - \hat{Y}_i$ is the forecast error at time i , Y_i is the actual observation at time i , and \hat{Y}_i is the forecasted value at time i .

In the interpretation of the Mean Absolute Percentage Error values, the scores of < 10% are viewed as excellent, while scores 10% to 20% generally portray good forecasts. (Cetin & Yavuz ,2020).

The Bayesian Information Criterion (*BIC*) and Akaike's Information Criterion (*AIC*) Statistics were used to select the best model. They can be expressed as $AIC = -2\ln(\text{Likelihood}) + 2r$ and $BIC = -2\ln(\text{Likelihood}) + r \ln(T)$. According to Gathondu (2014), the AIC value increases with the proportion of the assumed number of model parameters (r), and its score is considered best when the resultant value is lowest. The value of T represents the set of all parameters.

3.4 Holt-Winters Model Theory

This is also referred to as the triple exponential smoothing and is useful in forecasting data sets where trend and seasonal patterns are present. The exponential smoothing includes trend smoothing, seasonal smoothing, and overall smoothing.

The coefficients α, β, γ are the three smoothing parameters, and p stands for the number of observations made during each seasonal cycle. Holt-Winters methods are classified as additive or multiplicative (Susan *et al.*, 2015).

The additive HW method is used when a time series trend is linear and has a seasonal pattern (Hyndman & Khandakar, 2008; Hyndman *et al.*, 2008). It may be appropriate for modeling some tax heads. However, as economic performance changes, the tax trend usually changes in a multiplicative manner. The estimate L_t represents the series level, b_t the trend, S_t the seasonal component, α the level smoothing parameter, β the trend smoothing parameter, γ the seasonality component smoothing parameter, while F_{t+m} will be the forecast for the m periods ahead, and t the index-denoting period in this method.

The multiplicative model is represented by the equations below:

$$F_{t+m} = (L_t + b_t m) S_{t-s+m} \text{ for } m = 1, \dots, M \quad (4)$$

where,

$$L_t = \alpha \frac{Y_t}{S_{t-s}} + (1 - \alpha)(L_{t-1} + b_{t-1})$$

$$b_t = \beta(L_t - L_{t-1}) + (1 - \beta)b_{t-1}$$

$$S_t = \gamma \frac{Y_t}{L_t} + (1 - \gamma)S_{t-s}$$

and Additive Holt-Winters model is represented as:

$$F_{t+m} = L_t + b_t m + S_{t-s+m} \text{ for } m = 1, \dots, M \quad (5)$$

where,

$$L_t = \alpha(Y_t - S_{t-s}) + (1 - \alpha)(L_{t-1} + b_{t-1})$$

$$b_t = \beta(L_t - L_{t-1}) + (1 - \beta)b_{t-1}$$

$$S_t = \gamma(Y_t - L_t) + (1 - \gamma)S_{t-s}$$

The suitable Holt-Winters smoothing factors (α , β , and γ) for the best-fit model were determined based on the size of the errors computed from the error metrics.

3.5 SARIMA Model Theory

The Seasonal Autoregressive Integrated Moving Average (SARIMA) model was proposed by Hyndman and Khandakar (2008) and Box *et al.* (2015) to deal with non-stationary time series that exhibit seasonality. SARIMA models account for the seasonality component and occurrences of time series models. Seasonal differencing removes seasonality in a time series that is not stationary.

For instance, $Z_t = X_t - X_{t-s}$ where Z_t is the seasonally differenced series, and s is the number of seasons per year, which defines a first-order seasonal difference. This is the difference between an observation and a comparable observation from the previous season (Pongdatu & Putra, 2018).

SARIMA is denoted as ARIMA $(p, d, q) (P, D, Q)_s$

where,

(p, d, q) and $(P, D, Q)_s$ are the non-seasonal and seasonal parts of the model, respectively

Furthermore, the seasonal AR order is P , the seasonal MA order is Q , and the seasonal differencing is D .

The SARIMA model will be expressed as follows;

$$\Phi_p(B^s)\varphi(B)\nabla_s^D\nabla^d Z_t = \Theta_Q(B^s)\theta(B)\varepsilon_t \quad (6)$$

Considering that;

$\varphi(B)$ and $\theta(B)$ are the autoregressive and moving average polynomials of orders p and q .

$\Phi_p(B^s)$ and $\Theta_Q(B^s)$ are the seasonal autoregressive and moving average components with orders P and Q

∇^d and ∇_s^D are the ordinary and seasonal difference components, and B is the backshift operator.

$Z_t = (1 - B^d)(1 - B^s)^D m_t$ -represents the product of seasonal differencing D

ε_t is the non-stationary time series

s is the number of periods per season

The SARIMA model is further deconstructed by Chang and Liao (2010) as follows:

$\varphi(B) = 1 - \varphi_1 B - \varphi_2 B^2 - \dots - \varphi_p B^p$; the Non-seasonal *AR* of order p

$\Phi_p(B^s) = 1 - \Phi_1(B^s) - \Phi_2(B^{2s}) - \dots - \Phi_p(B^{ps})$; the Seasonal *AR* of order P

$\theta(B) = 1 + \theta_1 B + \theta_2 B^2 + \dots + \theta_q B^q$; the Non-seasonal *MA* of order q

$\Theta_Q(B^s) = 1 + \theta_1(B^s) + \theta_2(B^{2s}) + \dots + \theta_Q(B^{Qs})$ –Seasonal *MA* order Q

$$\nabla^d = (1 - B)^d$$

$$\nabla_s^D = (1 - B^s)^D$$

The research will concentrate on a 12-month revenue collection time series. This implies that the seasonal period is 12 ($s=12$).

As a result, the SARIMA model will be;

$$\Phi_p(B^{12})\varphi(B)\nabla_{12}^D\nabla^d Z_t = \Theta_Q(B^{12})\theta(B)\varepsilon_t \quad (7)$$

CHAPTER FOUR

RESULTS

4.1 The main contributors to the Domestic Taxes Department (DTD) annual tax revenues in Kenya

4.1.1 Descriptive Summary

Table 4.1: Major Tax Heads in Domestic Tax Summary Statistics

Statistics	Paye	Value	Corporation	Withholding	Excise	Total
		Added Tax	Tax	Tax		
Mean	19106773698	10227044967	12263661307	6111990845	5904013489	6.1175E+10
Standard Error	638320921.5	231914464.9	1508541596	289629122.1	425473991.1	2788343977
Median	20489223811	10375724537	6186943778	6354535705	6886116429	6.3026E+10
Standard Deviation	5637497735	2048213095	13323078633	2557935147	3757684544	2.4626E+10
Sample Variance	3.17814E+19	4.19518E+18	1.77504E+20	6.54303E+18	1.41202E+19	6.0644E+20
Kurtosis	0.1394	0.4454	-0.6698	-0.4032	-1.0216	-0.6249
Skewness	-0.8223	-0.6042	0.8907	-0.3650	-0.5246	-0.1893
Range	24324134831	9485496332	46604677098	9947997541	12016358262	9.3565E+10
Minimum	5412583113	4888093918	101483864	1136849737	0	1.3338E+10
Maximum	29736717944	14373590250	46706160962	11084847278	12016358262	1.069E+11
Sum	1.49033E+12	7.9771E+11	9.56566E+11	4.76735E+11	4.60513E+11	4.7716E+12
Count	78	78	78	78	78	78

The proportions of revenue contributed by the major tax heads are highlighted for the period between July 2014 to December 2020.

The average amount of revenue collected for Pay As You Earn was ($M=19,106,773,698$, $S.D=5637497735$), Value Added Tax ($M=10,227,044,967$, $S.D=2,048,213,095$), Corporation tax company ($M=12,263,661,307$, $S.D=13323078633$), Withholding tax ($M=6,111,990,845$, $S.D=2557935147$), Excise ($M=5,904,013,489$, $S.D=3757684544$) and Total amount ($M=6.1175E+10$, $S.D=2.4626E+10$) for the 78 months of interest.

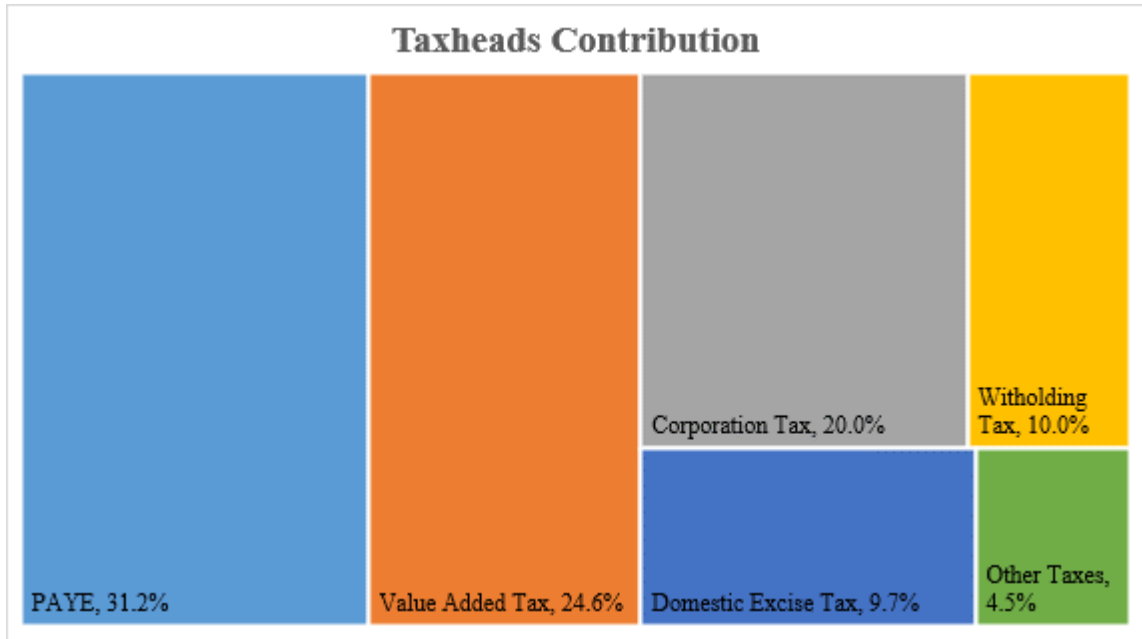


Figure 4.1: Tax Heads Contribution Tree Map

According to the analysis, 5 major tax heads were noted for the domestic taxes category. Pay As You Earn contributed the highest proportion of the total revenue (31.2%), followed by Value Added Tax (24.6%), Corporation tax (20%), withholding tax (10%), Domestic Excise (9.7%), and other taxes (4%). The other tax category comprises personal income taxes and agency revenues.

4.1.2 Regression and ANOVA Analysis of Proportion of Revenues Contributed.

Hypothesis Testing

Null Hypothesis (Ho): The proportions of revenue contributed by various taxes in Kenya are the same.

Alternative Hypothesis (Ha): The proportions of revenue contributed by various taxes in Kenya are significantly different.

The regression analysis was used to investigate whether the revenue contributed by various tax heads significantly differed. The major tax heads were involved.

Table 4.2: Regression Model

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	0.996 ^a	0.992	0.991	2302319209.5395

a. Predictors: (Constant), Pay As You Earn, Corporation, Value Added Tax, Excise, Withholding Tax

The R² value is 0.992. This suggests that the major tax heads involved in the model explain about 99.2% of the variations in the model. The remaining 0.8% of the variation is attributed to external forces and other factors not included in the model.

Table 4.3: ANOVA Analysis

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	4.631E+22	5	9.263E+21	1747.483	.000 ^b
	Residual	3.816E+22	72	5.301E+18		
	Total	4.670E+22	77			

A. Dependent Variable: Total

B. Predictors: (Constant), Pay As You Earn, Corporation Tax, Value Added Tax, Excise, Withholding tax

The ANOVA analysis (Table 4.3) revealed a $F(5,72)=1747.48$, $p=.000$). The p-value is less than .05 hence, the null hypothesis is rejected. This reveals a significant influence of the Pay As You Earn, Corporation Tax, Value Added Tax, Excise Tax, and Withholding Tax on the total Domestic tax collection. This also implies that the proportion of revenue contributed by various taxes in Kenya significantly differs.

4.2 Forecasting of Kenya's Domestic Taxes annual tax revenues using SARIMA and Holt-Winters time series models.

4.2.1 Domestic Tax Revenue Trend

Between July 2014 and December 2020, the total DTD collection revealed an increasing and fluctuating trend. The highest amounts of tax collections were noticed at the middle and end of every year. This was highly attributed to the installment tax and the balance of tax. The major due dates for installment taxes fall in April, June, September, and December of each year, while the balance of tax is mainly paid by 30th June of every year. Similarly, in December, an upward trend was witnessed due rise in value-added tax that

is attributed to the consumption of VATable products as the year ends due to festivities. This is illustrated in the Figure below.

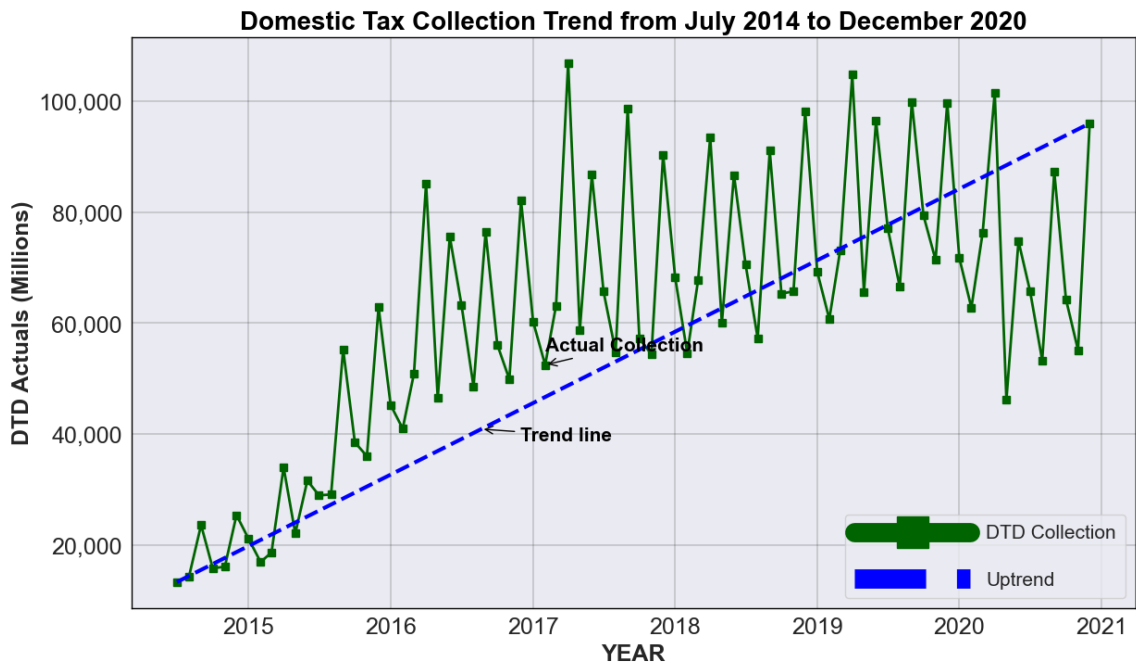


Figure 4.2: Time plot of Domestic Taxes from July 2014 to December 2020

Figure 4.2 above indicates the presence of variability and seasonality, hence the need for the stationarity test.

4.2.2 Stationarity Test

It is recommended to test for stationarity before using a variable with time series data for modeling to avoid estimates of misleading relations. To reduce data variability and seasonality, differencing was done ($y' = y_t - y_{t-1}$), as shown in Figure 4.3.

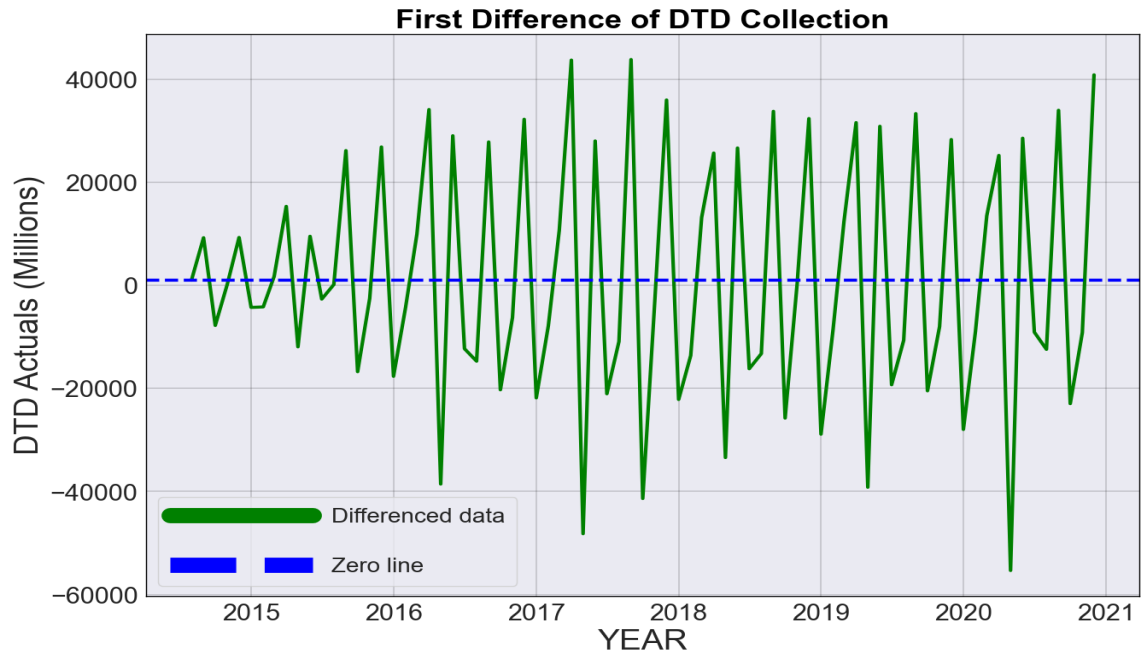


Figure 4.3: Domestic Tax Trend at First Difference

Figure 4.4 represents the detrended and deseasonalized data. This allowed the isolation of trends and seasonal patterns, hence making more accurate forecasts.

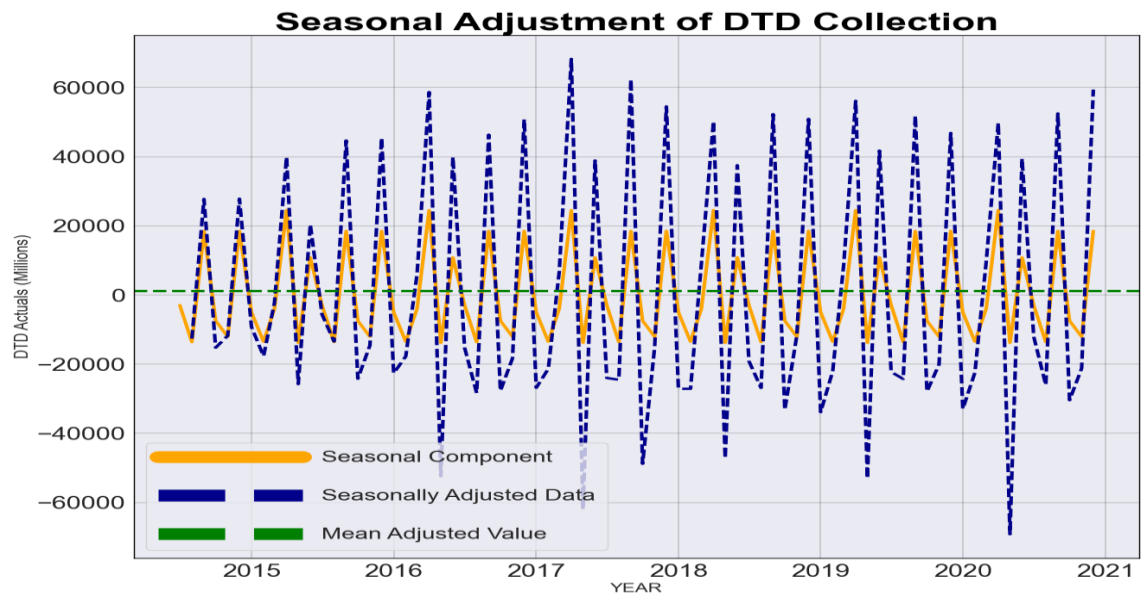


Figure 4.4: Detrended and Deseasonalized data

The stationarity was also confirmed using Phillips-Perron (PP) and Augmented Dickey-Fuller (ADF) tests. The assumption in the Null hypothesis is that the time series is non-stationary.

Table 4.4: Testing for Stationarity

Level	P-values		Decision
	ADF Test	PP Test	
Time series at level	0.8395	0.00000148	Not stationary and Stationary
First Difference	0.01	0.000000183	Both stationary
Log First Difference	0.01	0.000000899	Both stationary

When transformed and differenced once, the ADF and PP tests resulted in ($ADF = -6.89, p = 0.01$) and ($F(1,74) = 33.13, p < 0.05$) respectively. A p-value of less than 5% indicates that the series is stationary since the null hypothesis is rejected, as shown in Table 4.4.

Decomposition of Time Series Data into Its Components

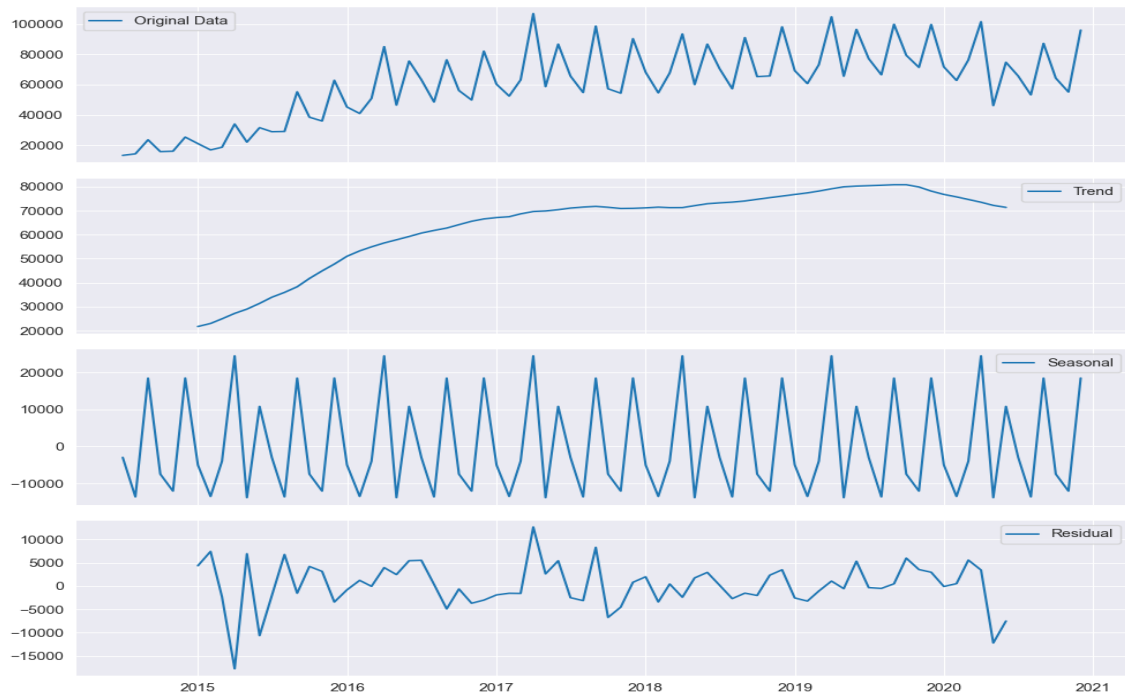


Figure 4.5: Data Decomposition

The time series data was decomposed into random, seasonal, trend, and observed patterns. This helps provide a clear understanding of the data, making analysis and forecasting easier and more accurate.

4.2.3 SARIMA Model

4.2.3.1 SARIMA Model Selection

Several models were fitted by analyzing the time series data, and the optimal model was identified based on information criteria methods. The diagnostic checks were then performed to assess the adequacy of the selected model. The best model was determined by identifying the most suitable combination of parameters for both the non-seasonal and seasonal components of the model.

Table 4.5 provides a comparative overview of several SARIMA models fitted to a time series with a seasonal period of 12. The models were evaluated using three information criteria: AIC, AICc, and BIC. Among the eleven iterated models, the ARIMA (0,1,1) (0,1,0)₁₂ had the lowest values for AIC (1358.683), AICc (1358.876), and BIC (1363.031), indicating it was the best-fitting model for the domestic tax data. The best model values are as follows:

p = 0	d = 1	q = 1	P = 0	D = 1	Q = 0	m = 12
non seasonal AR = 0	non seasonal differencing = 1	non seasonal MA = 1	non seasonal AR = 0	seasonal differencing = 1	seasonal MA = 0	seasonal period= 12

The model consistently achieved the lowest values across the three information criteria, hence providing strong evidence for its selection. The proximity of the AIC and AICc values of all the selected models signifies that the sample size was adequately chosen and no overfitting occurred. This also implies that less information is lost when predicting using this model.

Table 4.5: SARIMA $(p, d, q)(P, D, Q)_{12}$ models

Rank	Model	AIC	AICc	BIC
1	ARIMA(0,1,1)(0,1,0)[12]	1358.683	1358.876	1363.031
2	ARIMA(0,1,1)(0,1,1)[12]	1359.075	1359.469	1365.598
3	ARIMA(0,1,1)(1,1,0)[12]	1359.472	1359.866	1365.995
4	ARIMA(0,1,2)(0,1,0)[12]	1360.445	1360.838	1366.968
5	ARIMA(1,1,1)(0,1,0)[12]	1360.556	1360.949	1367.079
6	ARIMA(0,1,1)(1,1,1)[12]	1360.889	1361.556	1369.586
7	ARIMA(1,1,2)(0,1,0)[12]	1361.396	1362.063	1370.094
8	ARIMA(2,1,2)(1,1,1)[12]	1363.884	1365.849	1379.104
9	ARIMA(1,1,0)(0,1,0)[12]	1364.376	1364.57	1368.725
10	ARIMA(1,1,0)(1,1,0)[12]	1365.142	1365.536	1371.665
11	ARIMA(0,1,0)(0,1,0)[12]	1374.551	1374.614	1376.725

4.2.3.2 Diagnostic Test for SARIMA

The diagnostic checks were carried out using the residuals from the models. The study used ACF plots of the residuals, Histograms, and p-values from the Ljung-Box statistic. The histogram plot compares the distribution of residuals to the normal distribution (yellow line) and the kernel density estimate (green line). The histogram and kernel density were found to follow the normal curve (yellow line), forming a bell-shaped appearance. This implies that the normality assumption was met by the data, as illustrated in Figure 4.6.

The normal Q-Q plot compared the resultant quantiles from the residuals to those from the normal distribution. Most points fall along the 45-degree line of best fit, which is an indicator that the model residuals are normal or close to the normal distribution. There exist a few points away from the reference line, but not extreme, which is common in most datasets.

The correlogram (Figure 4.6) displayed the autocorrelation of the residuals. Most points were found within the confidence bound. This suggests that no autocorrelation was within the residuals. The ACF from the SARIMA residuals was very near the zero line, with

practically most spikes falling within the significant zone. This demonstrates the independence of the residuals (Hidayat, Ilyas, & Yuliani, 2024). This suggests that the model successfully picked up any of the time-dependent patterns that were found in the data. Two lags are observable, which is acceptable for this analysis. The model, therefore, captured all the temporal dependencies well.

This is also confirmed by the Ljung-Box test ($\chi^2_{15} = 15.12, p = 0.4428$), which suggests the independence of residuals.

The SARIMA diagnostic checks confirmed that the residuals were uncorrelated, emanated from a well-specified model, and could be utilized to forecast.

ARIMA(0,1,1)(0,1,0)[12] DTD Model Diagnostics

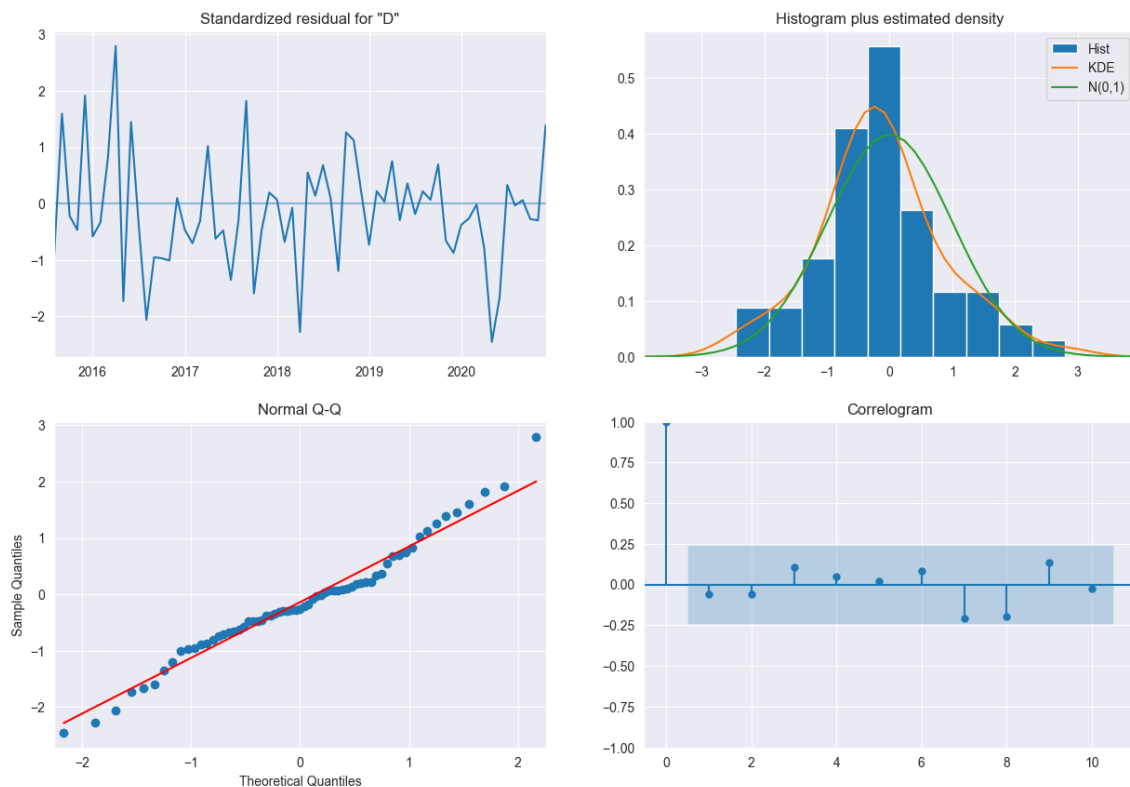


Figure 4.6: SARIMA Model diagnostics

4.2.3.3 SARIMA Forecast

The observed domestic tax collections between July 2014 and December 2020 were used to predict the likely collection in the following 12 months of the year 2021.

Table 4.6 shows the monthly average forecasts with the 80% to 95% bounds for the year 2021. In each of the bounds, there is an estimate for the upper and lower bounds of the two intervals. The estimates vary significantly across the months, with peaks in April (Ksh. 91,701.08), September (Ksh. 77,415.74), and December (Ksh. 86,166.77), which are the months where more domestic taxes are collected. On the upper bound, the peak months resulted in April (112,069.05), September (Ksh. 103,494.01), and December (Ksh. 115,136.31). The all scenarios, May and August were the months where the lowest forecasts were recorded, reflecting months of lower expectations in domestic taxes in the forecast period.

Table 4.6: SARIMA Forecasted Estimates (Ksh. Millions)

Month	Point Forecast	Lo 80	Hi 80	Lo 95	Hi 95
Jan-21	61,892.05	51,437.25	72,346.85	45,902.82	77,881.28
Feb-21	52,949.75	41,461.29	64,438.22	35,379.66	70,519.84
Mar-21	66,468.11	54,031.59	78,904.62	47,448.10	85,488.11
Apr-21	91,702.08	78,384.84	105,019.33	71,335.12	112,069.05
May-21	36,352.80	22,209.56	50,496.03	14,722.59	57,983.01
Jun-21	64,930.93	50,007.35	79,854.51	42,107.29	87,754.57
Jul-21	55,846.48	40,181.39	71,511.58	31,888.78	79,804.18
Aug-21	43,419.06	27,045.99	59,792.13	18,378.62	68,459.50
Sep-21	77,415.74	60,364.07	94,467.41	51,337.46	103,494.01
Oct-21	54,453.30	36,749.03	72,157.58	27,376.95	81,529.66
Nov-21	45,292.45	26,958.77	63,626.12	17,253.52	73,331.38
Dec-21	86,166.77	67,224.60	105,108.93	57,197.23	115,136.31

Figures 4.7 below illustrate how the SARIMA model projections were visualized. The forecast line (blue dotted line) outlines the expected fluctuations in domestic revenues in a period of 1 year. The 95% confidence interval highlights the progressive uncertainty following the extended horizon of the forecast. It remained aligned with the previous seasonal trend. This implies that the model has adequately learned from the historical data for the short-term forecast.

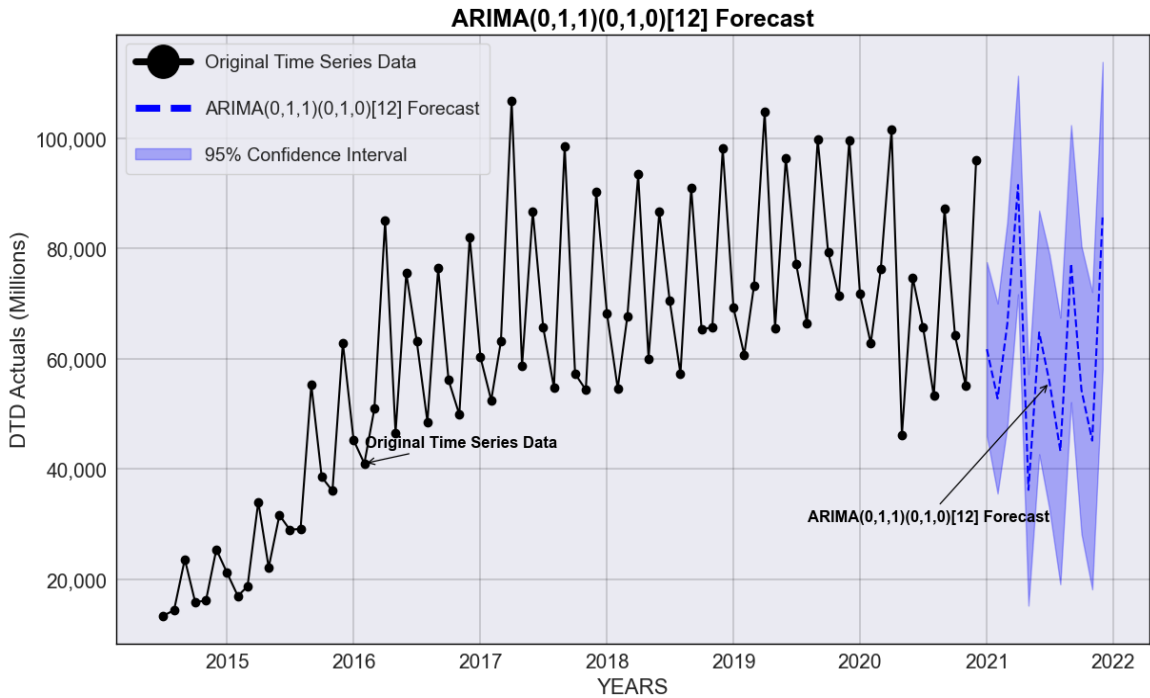


Figure 4.7: SARIMA DTD Forecast

4.2.4 Holt-Winters Model

HW exponential smoothing technique was applied to forecast the domestic taxes. This model is typically applied when a time series exhibits seasonality and a linear trend. Therefore, one can use the original data with seasonality and internally account for the seasonal effects (Holt, 1957). Using the weighted averages to update the level, trend, and seasonal components enables it to effectively capture evolving patterns within the data.

4.2.4.1 Holt-Winters Model Selection

This method categorizes the study data into three parts or components: trend, level value, and seasonal, with weights ranging from 0 to 1 to enable model fitting and prediction. The Holt-Winters model finds the optimal smoothing factors ($\alpha=\alpha$, $\beta=\beta$, and $\gamma=\gamma$). Using the smoothing factors, the additive and multiplicative models are represented using the formula;

$$Forecast_t = Level + Trend + Seasonality$$

Table 4.7: Holt-Winter smoothing factors

		Additive- Holt-Winters	Multiplicative- Holt-Winters
Factors	alpha(α)	0.523	0.679
	beta(β)	0.054	0.071
	gamma(γ)	0.965	1
	Level Mean(a)	76365.890	72192.81
	Trend(b)	185.665	-131.98
Smoothing Parameters	\hat{S}_1	-10052.874	0.95
	\hat{S}_2	-17426.104	0.81
	\hat{S}_3	-3734.406	0.93
	\hat{S}_4	23813.634	1.26
	\hat{S}_5	-22338.760	0.69
	\hat{S}_6	9224.005	1.10
	\hat{S}_7	-4676.475	0.90
	\hat{S}_8	-18829.459	0.74
	\hat{S}_9	14722.804	1.23
	\hat{S}_{10}	-9001.395	0.93
	\hat{S}_{11}	-16775.452	0.83
	\hat{S}_{12}	19499.240	1.33

The resultant additive Holt-Winters model for domestic tax collections (Y_t), level mean estimate (L_t), trend estimate (b_t), and seasonal factors (S_t) are as follows:

$$\ln(Y_t) = (76365.89 - 185.55t) + s_t + e_t$$

$$b_t = 0.054(L_t - L_{t-1}) + (1 - 0.054)b_{t-1}$$

$$S_t = 0.965(Y_t - L_t) + (1 - 0.965)S_{t-1} \approx 0.965(Y_t - L_t) + 0.035S_{t-1}$$

The resulting multiplicative Holt-Winters model, on the other hand, can be fitted as indicated below;

$$Y_t = (72192.81t) + sn_t + e_t$$

$$L_t = 0.679\left(\frac{Y_t}{S_{t-s}}\right) + (1 - 0.679)(L_{t-1} + b_{t-1}) \approx 0.679\left(\frac{Y_t}{S_{t-s}}\right) + 0.321(L_{t-1} + b_{t-1})$$

$$b_t = 0.071(L_t - L_{t-1}) + (1 - 0.071)b_{t-1} \approx 0.071(L_t - L_{t-1}) + 0.929b_{t-1}$$

$$S_t = 1\left(\frac{Y_t}{L_t}\right) + (1 - 1)S_{t-s} \approx 1\left(\frac{Y_t}{L_t}\right)$$

4.2.4.2 Diagnostic Test for Holt-Winters

In the Additive Holt-Winters model, the residuals were mostly centered around zero and fell within a moderate range. However, some extreme values were observed, being less than -15,000 and greater than 15,000. The ACF plot shows no significant autocorrelation in the residuals, as most spikes lie within the 95% confidence interval (blue shaded area). For the Multiplicative Holt-Winter model, residuals were close to zero and approximately normally distributed, though the distribution is slightly positively skewed. This skewness may indicate occasional underestimation by the model or the possible presence of some outliers. The ACF plot outlines most of the points within the 95% confidence bounds, which is an indication of the absence of autocorrelation at almost all lags. Holt-Winters diagnostic checks confirmed that the residuals were uncorrelated, emanated from a well-specified model, and could be utilized to forecast.

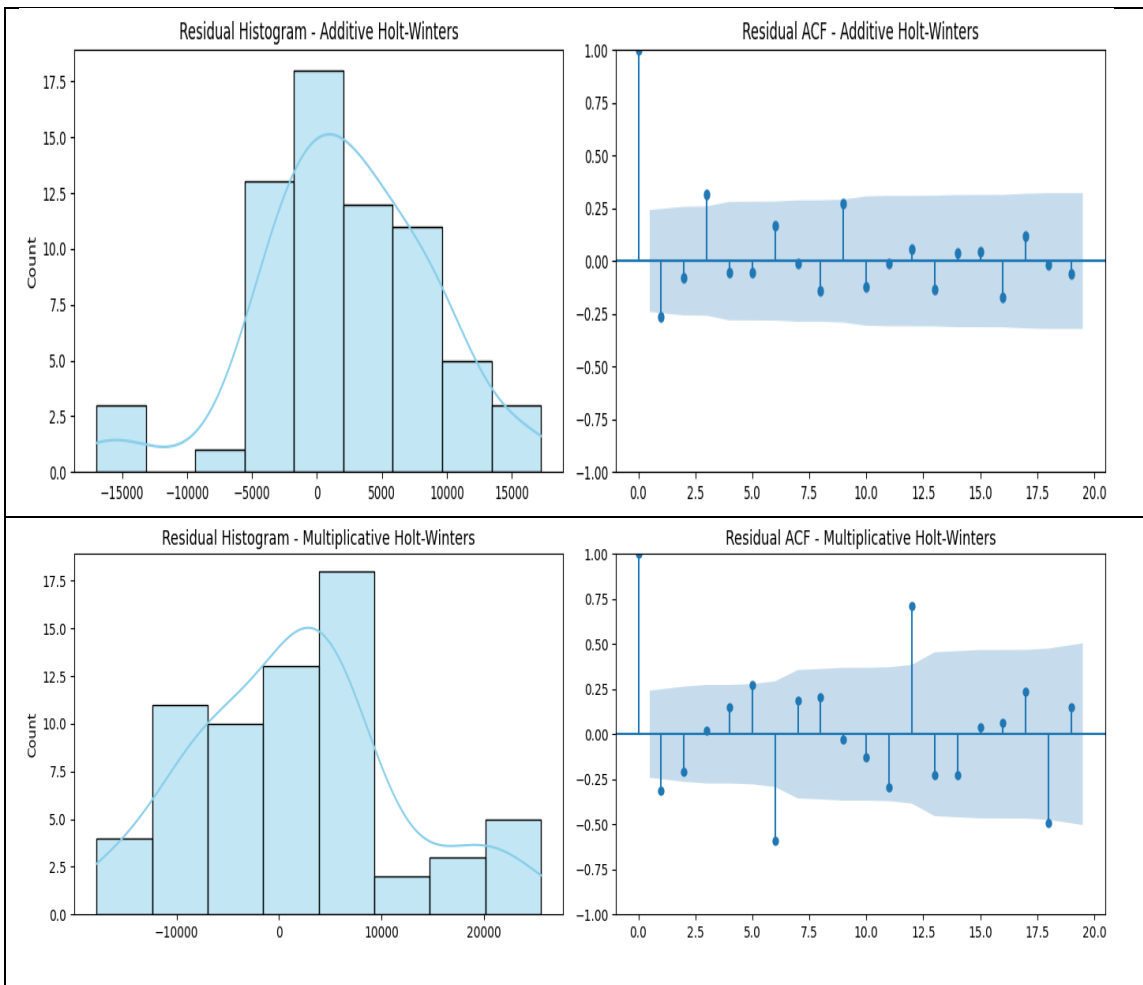


Figure 4.8: HW Diagnostic tests

4.2.4.3 Holt-Winters Forecast

The observed domestic tax collections between July 2014 and December 2020 were used to predict the likely revenues collected in the following 12 months of the year 2021. Figures 4.9 and 4.10 below illustrate how the Additive and Multiplicative Holt-Winters model projections were visualized.

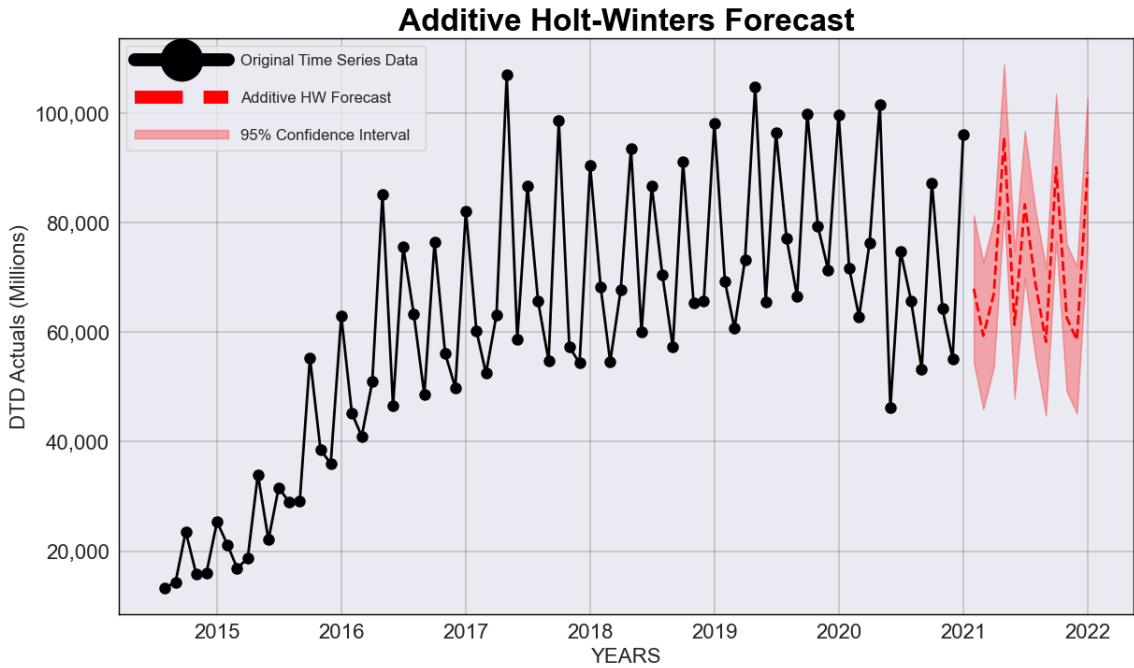


Figure 4.9: Additive HW Forecast

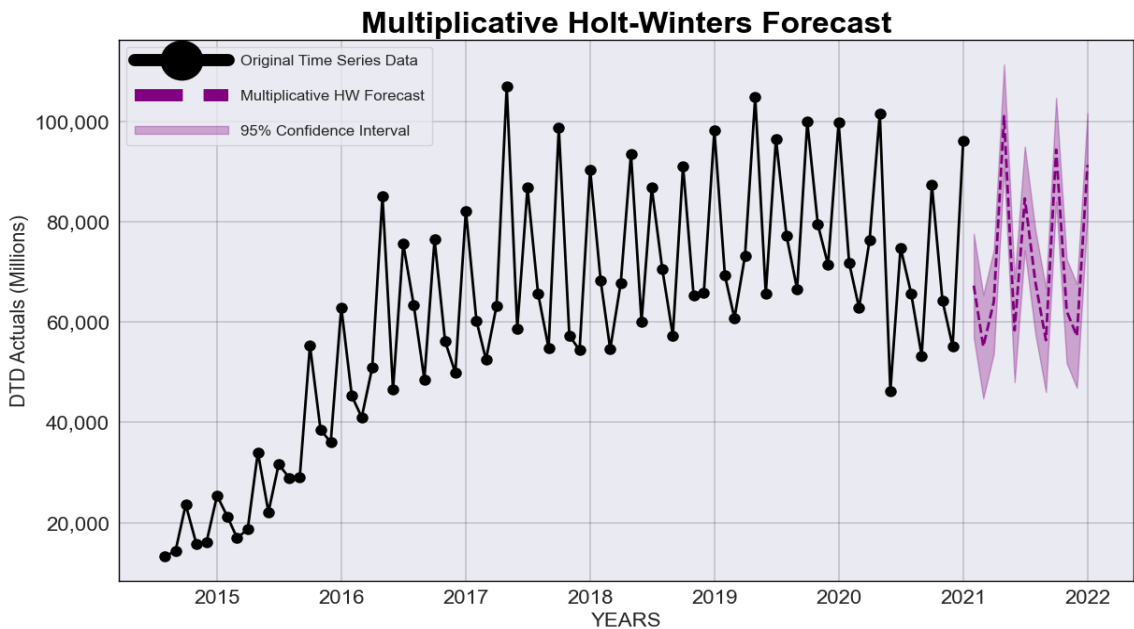


Figure 4.10: Multiplicative HW Forecast

The pattern of the forecasted part (January 2021 to December 2021) was almost similar to the actual. This shows that the models were good at forecasting domestic tax data. In Figures 4.9 and 4.10, the trend line (black) outlines the past domestic taxes data from July 2014 to December 2020. The red and purple sections show predicted values of the domestic tax collections from January 2021 to December 2021. The one-year forecast. This displays the smoothed extension of the historical data of the domestic taxes recorded in Kenya. The confidence intervals around the predicted area indicate the likely variability of the one-year forecast period. The bounds of the interval are not so wide, implying that the uncertainty levels are not so huge.

4.3 The best model for forecasting the Domestic tax revenues in Kenya

4.3.1 Comparative Evaluation of the Model's Accuracy

The resultant prediction models were tested to deduce the model with the least prediction errors (MAE, MAPE, and MASE). The SARIMA (0,1,1)(0,1,0)₁₂ model had the least errors, followed by the multiplicative and additive Holt-Winters models, respectively (Table 4.8). In all the models, MAPE values were less than 10%, signifying excellent model forecasts.

Table 4.8: Holt-Winters and SARIMA Mode Error Metrics

Models	SARIMA (0,1,1)(0,1,0)₁₂	Additive Holt-Winters Method	Multiplicative Holt- Winters Method
MAE	4998.15	6108.80	5206.61
MAPE	7.67	9.992	8.075
MASE	0.38	0.46	0.39

4.3.2 Prediction versus Actual Data

The domestic tax total collections for 12 months were predicted using the 3 test models and compared with the actual data from the revenue agency. These collections were aggregated to fit the financial calendar of the country, which depicts the three months' total collection for each quarter. The resultant values are shown in Table 4.9. The above table compares the performance of the of the three times series forecasting models;

Additive HW, Multiplicative HW, and SARIMA against the actual observed values for the four quarters in the financial year. The RMSE and MAE error metrics were used.

Table 4.9: Domestic taxes Predicted and Actual Values in Ksh. Millions.

Period	Additive HW Values	Multiplicative HW Values	Sarima Values	Actual Values
Q3	253,170.30	243,497.87	233,889.23	208,307.79
Q4	316,379.22	302,343.99	257,806.63	283,721.06
Q1	318,065.84	314,997.12	251,757.69	265,190.88
Q2	342,118.04	364,852.70	269,997.35	280,193.16
RMSE	16838.15	18670.48	8219.96	
MAE	16026.71	15689.90	6432.64	

Source: Authors' Computation

The comparison above provided insights into the model that resulted in the least errors according to the laid selection criteria. This was the simulation with the lowest RMSE and MAE values. SARIMA model produced the most accurate forecasts, achieving the lowest RMSE (8,219.96) and MAE (6,432.64). This was followed by the Additive Holt-Winters model (RMSE = 16,838.15, MAE = 16,026.71) and the Multiplicative Holt-Winters model (RMSE = 18,670.48, MAE = 15,689.90).

The Additive and Multiplicative Holt-Winters methods exhibited slightly higher error metrics and therefore deviated more from the actual values. While the Multiplicative HW method had a marginally better MAE, it performed worse in terms of RMSE, suggesting that the seasonal pattern in the data was not strongly multiplicative. For all models, the estimates were above the actual values in Quarter 3 and Quarter 2. SARIMA's predictions remained consistently closer to the actual values across all four quarters.

4.3.3 Model Evaluation

4.3.3.1 Diebold-Mariano (DM) test

The test was applied on the three test models to deduce whether indeed they were significantly different. The method evaluates whether the accuracies deferred, based on the variances of models pair loss differentials. In this case, initial model assumes that the predictive accuracy of the model pair is similar and vice versa, which is assessed through

analysis of the loss differential series derived from their forecast errors (Mohammed & Mousa, 2020) The DM test is important because it confirms whether the model selected based on simple error metrics, such as RMSE and MAE, is genuinely superior to the others for the seasonal domestic revenue data. Additionally, since forecast errors may exhibit autocorrelation, the DM test adjusts for this issue, providing a more robust and reliable inference regarding the model's comparative accuracy.

The DM test is expressed as:

$$DM = \frac{\bar{d}}{\sqrt{\frac{\hat{V}(d)}{T}}}; \bar{d} \text{ is the average of the loss differentials,}$$

$\hat{V}(d)$ Variance of loss differentials of the observations.

Hypothesis for Diebold-Mariano (DW) Test

Taking Model 1=SARIMA, Model 2=Additive HW, and Model 3=Multiplicative HW, below is the DM test hypothesis for each pair of models.

1. SARIMA vs Additive HW (DW Test 1)

Null Hypothesis (Ho): The two models have equal predictive accuracy

$$(H_0: E[Loss(e_{1,t}) - Loss(e_{2,t})] = 0)$$

Alternative Hypothesis (H_a): The two models have no equal predictive accuracy

$$(H_a: E[Loss(e_{1,t}) - Loss(e_{2,t})] \neq 0)$$

2. SARIMA vs Multiplicative HW (Test 2)

Null Hypothesis (Ho): The two models have equal predictive accuracy

$$(H_0: E[Loss(e_{1,t}) - Loss(e_{3,t})] = 0)$$

Alternative Hypothesis (H_a): The two models have no equal predictive accuracy

$$(H_a: E[Loss(e_{1,t}) - Loss(e_{3,t})] \neq 0)$$

3. Additive HW vs Multiplicative HW (Test 3)

Null Hypothesis (Ho): The two models have equal predictive accuracy

$$(H_0: E[Loss(e_{2,t}) - Loss(e_{3,t})] = 0)$$

Alternative Hypothesis (H_a): The two models have no equal predictive accuracy

$$(H_a: E[Loss(e_{2,t}) - Loss(e_{3,t})] \neq 0)$$

The results of the analysis are provided in Table 4.10 as follows:

Table 4.10: Diebold-Mariano (DM) Test Results

Model Pair	DM Value	P-value
DM Test 1:SARIMA Vs Additive HW	-3.499	0.00498
DM Test 2:SARIMA Vs Multiplicative HW	-1.972	0.07425
DM Test 3:Additive HW Vs Multiplicative HW	-0.5368	0.6021

The test statistics for the Diebold-Mariano (DM) tests comparing the three model pairs were: DM = -3.499 (p < .05), DM = -1.972 (p = 0.074), and DM = -0.537 (p = 0.602), respectively. Since the p-value for the first test is less than .05, we reject the null hypothesis of equal predictive accuracy for that pair (Diebold & Mariano, 2002). This indicates that Model 1 (SARIMA) has significantly better predictive accuracy compared to its counterpart. For the second test, the p-value of .074 suggests a marginal difference between SARIMA and the Multiplicative Holt-Winters model, with SARIMA likely performing better, though this difference is insignificant. The p-values for the second and third tests are greater than .05, indicating no statistically significant difference in predictive accuracy between SARIMA and Multiplicative Holt-Winters, and between Additive and Multiplicative Holt-Winters, respectively. The negative DM statistics of -3.499 and -1.972 (as shown in Table 4.10) indicate that Model 1 had lower forecast errors than the models it was compared against. Similarly, the DM value of -0.537 suggests that the Additive Holt-Winters method had slightly smaller errors compared to the Multiplicative Holt-Winters method, although this difference is not statistically significant.

4.3.3.2 Model confidence set (MCS) Procedure

This procedure was applied to the three model pairs to eliminate inferior models. This method is used to identify models that are statistically indistinguishable in terms of predictive accuracy, as well as to exclude models that are statistically worse than the best-performing ones. Using the MCS procedure at a 5% significance level, Model 1 (SARIMA) was retained in the final model set (p=0.027), making it the only superior

model. Models 2 (Additive Holt-Winters) and 3 (Multiplicative Holt-Winters) were excluded from the set, indicating that they had lower predictive accuracy compared to SARIMA and were slightly statistically inferior.

Table 4.11: Model Confidence Set (MCS) Test Results

Model	Rank	Test Statistic	Indicator for inclusion in the MCS	Loss Function	p-value
mse1	1	-2.1551	1	67,567,723	0.028

CHAPTER FIVE

DISCUSSION, CONCLUSION, AND RECOMMENDATIONS

5.1 Summary

The principal aim of this paper was to use time series techniques (SARIMA and Holt-Winters) to forecast domestic tax revenue collected in Kenya for a duration of 78 months. The background section outlines the general overview of the research work, which mentions the crucial mandate of taxation towards the mobilization of funds that are used in a country's national development. In this section, a summary of some of the forecasting methods commonly used in various parts of the world. The literature review outlines the previous research that has been done in various sectors and countries. It also assesses the results of these studies and the gaps identified in the forecasting of tax revenues.

The materials and methods section focuses on the data source and the period of data collection, as well as the study design of the forecasting process. The measures used to test the accuracy and perform diagnostic checking are well explained, including how they are analyzed. The data preparation and scrutiny techniques applied to make it usable for forecasting were also mentioned. Extensive theory and the equations that form the basis of the time series techniques are well highlighted.

The results section outlines the summary statistics of the revenues, differencing and detrended figures, trend analysis, and the test of stationarity. It also outlines the outcome of the prediction in tables and figures, together with the error metrics. It also includes the summaries of the insights from these results.

5.2 Discussion

In this study, the secondary data on domestic taxes for 78 months was collected and used to make inferences on the forecasting abilities of the SARIMA and HW techniques to fit a suitable model that could be used in predicting future revenue collections. This was inspired by the important role played by exchequer revenues in national development in the country and the quest to develop alternatives to accurately model these revenues and produce realistic estimates.

One of the primary aims of the study was to investigate the main contributors to the total annual domestic tax revenues collected in Kenya between July 2014 and December 2020. Pay As You Earn was found to be the highest contributor, contributing to about a third of the total tax during the period. This was followed by the Value Added Tax, Corporate Income Tax, Withholding Tax, and Domestic Excise Tax. These tax heads contributed approximately 95.5% of the total revenue collected by the domestic tax department during the study period. These findings highlight priority areas where the government could focus revenue mobilization efforts by fostering an enabling environment for stakeholders and businesses to thrive, thereby increasing revenue collection.

A comparative evaluation of the three forecasting methods to assess their predictive performance identified the SARIMA (0,1,1)(0,1,0)₁₂ model as the most effective. Although the additive and multiplicative Holt-Winters methods proved adequate, they were inferior to SARIMA. The SARIMA model exhibited the lowest prediction errors across key metrics: MAE (4,998.15), MAPE (7.67%), and MASE (0.38). All three models achieved MAPE values below 10%, indicating excellent forecasting accuracy, with SARIMA consistently producing the smallest errors.

Further comparison using actual revenue data from the forecast period showed that SARIMA attained the lowest RMSE (8,219.96) and MAE (6,432.64) values across all quarters. While the multiplicative Holt-Winters model had a slightly better MAE than the additive version, it performed worse in terms of RMSE, suggesting that the seasonal pattern in the data was not strongly multiplicative.

To rigorously assess predictive accuracy, the Diebold-Mariano test was employed. The test favored SARIMA over the additive Holt-Winters model ($DM = -3.499$, $p < .05$) and showed a marginal but statistically insignificant advantage over the multiplicative Holt-Winters model ($DM = -1.972$, $p = 0.074$). The Model Confidence Set (MCS) procedure further confirmed SARIMA as the only statistically superior model at the 5% significance level ($p = 0.027$), excluding both Holt-Winters methods.

In summary, while Holt-Winters models performed well, SARIMA was the most reliable and accurate model for forecasting domestic tax revenues in this study, particularly in capturing both seasonal and trend components of the data.

The results align with extensive work done in various countries that point to SARIMA as a preferred choice in forecasting data with complex seasonal patterns and longer periods, whereas the Holt-Winters model tends to perform better on data with stable seasonal patterns. In forecasting export data in West Sumatra, Hasibuan *et al.* (2023) found the SARIMA model to have better forecasting proficiency in relation to the Holt-Winters model. It resulted in a lower MAPE value of 0.44% and MAD of 0.89% compared to the Holt-Winters model, indicating a higher accuracy in capturing the seasonal fluctuations. Similarly, in the forecasting of Nigeria's tax revenue, Tasi'u *et al.* (2024) found the SARIMA (3,2,1)(0,1,1)₄ model to have the lowest MAE and RMSE values (0.082 and 0.165) and outperformed the Holt-Winters model.

Makananisa (2015) reviewed the SARIMA and HW techniques in the prediction of key revenues together with the total collected taxes in South Africa using twelve periods data from 1995 to 2010. The study discovered that the two methods were suitable and had different capabilities in the prediction of different taxes. For instance, Personal Income Tax and Value Added Taxes were accurately estimated by SARIMA, while the HW method was superior in forecasting Corporate Income and Total Tax Revenue. These results confirmed that both methods are suitable and depend on the data traits and type of tax head. Additionally, Kithure (2018), in the forecasting of VAT in Kenya, using data from 2009 to 2016, found the ARIMA time series model to be suitable.

The results of the Autoregressive Integrated Moving Average model performed better compared to the Holt-Winters model in the forecasting of Value Added Tax in Ghana. (Makridakis *et al.*, 2008). ARIMA (1,1,4) had lower forecasting errors, hence produced a better model. This study result slightly differs from the outcome of this study.

In general, the outcomes of this research work provide a background that policymakers and government officers can use to develop realistic forecasts, which can aid in minimizing the fiscal deficits in the country. This adds to the literature in forecasting, more so of revenues in Kenya. SARIMA and Holt-Winters methods were evaluated and demonstrated to be sufficiently accurate models to estimate Kenya's domestic tax revenues. However, the SARIMA model was more suitable and had fewer forecasting errors compared to its counterparts.

5.3 Conclusion

The study was aimed at using domestic revenue data collected in Kenya between July 2014 to December 2020 to gauge the predictive abilities of the Additive, Multiplicative HW methods, and SARIMA models. Considering they are time series models, the Box-Jenkins Procedure was utilized. This included the identification of the superior model, estimation, review of the model diagnostics, and ultimately, forecasting. On the model diagnostic checking, the ACF, Ljung-Box statistics, and PACF test were examined and found satisfactory. Similarly, the tests for normality were met. This implied that the data was fit for the application of the time series techniques in estimation.

Main contributors of Domestic Taxes tax revenues in Kenya

The main contributors to domestic tax revenues in Kenya between July 2014 and December 2020 were identified based on the proportions of revenue collected within the period.

The analysis highlighted five key tax categories within domestic taxes. Pay as You Earn (PAYE) was the largest contributor, accounting for 31.2% of total revenue, with an average monthly collection of approximately Ksh. 19.1 billion KES (SD =Ksh. 5.64 billion). This was followed by Value Added Tax (VAT), contributing 24.6% of revenue, with an average of about Kshs. 10.2 billion KES (SD =Ksh. 2.05 billion). Corporation tax ranked third, making up 20% of the total, with an average monthly collection of 12.3 billion KES (SD=13.3 billion). Withholding tax and Domestic tax contributed 10% and 9.7% respectively, with an average collection of Ksh. 6.1billion (SD=Ksh. 2.56 billion) and Ksh. 5.8billion (Ksh. 3.76 billion).

The remaining 4% came from other taxes, which include personal income taxes and agency revenues. Overall, these five tax heads formed the bulk of domestic tax revenue, underscoring their critical role in Kenya's fiscal system.

Forecasting Kenya's Domestic tax revenues using SARIMA, and Holt-Winters time series models.

Between January and December 2021, all three models, SARIMA, Additive Holt-Winters, and Multiplicative Holt-Winters, predicted an upward trend in domestic tax revenues, with

December consistently showing the highest forecasted values. The total predicted revenue for the year was approximately Ksh. 1,013.4 million for SARIMA, Ksh. 1,229.7 million for Additive Holt-Winters, and Ksh. 1,225.6 million for Multiplicative Holt-Winters. This corresponds to average monthly projections of Ksh 84.45 million, Ksh 102.48 million, and Ksh. 102.14 million, respectively.

Quarterly forecasts from the SARIMA model were Ksh 233.89 million for Q3, Ksh. 257.81 million for Q4, Ksh. 251.76 million for Q1, and Ksh. 269.99 million for Q2. The Multiplicative Holt-Winters model predicted Ksh. 243.50 million for Q3, Ksh. 302.34 million for Q4, Ksh. 315.00 million for Q1, and Ksh. 364.85 million for Q2. The Additive Holt-Winters model forecasted Ksh. 253.17 million for Q3, Ksh. 316.38 million for Q4, Ksh. 318.07 million for Q1, and Ksh. 342.12 million for Q2.

Comparing these forecasts to actual revenues:

In Q1, both Holt-Winters models overestimated the actual revenue of Ksh. 265.19 million, while SARIMA underestimated it.

In Q2, the Multiplicative Holt-Winters model significantly overestimated revenue at Ksh. 364.85 million compared to the actual Ksh. 280.19 million; Additive Holt-Winters also overestimated, whereas SARIMA's forecast was closer but was still below the actual value.

In Q3, all models overestimated revenue, with Additive Holt-Winters having the highest forecast (Ksh. 253.17 million) and SARIMA the lowest (Ksh. 233.89 million), against an actual of Ksh 208.31 million.

In Q4, both Holt-Winters models over predicted revenue, while SARIMA underestimated it relative to the actual Ksh. 283.72 million; Additive Holt-Winters forecasted Ksh. 316.38 million, Multiplicative Holt-Winters Ksh. 302.34 million, and SARIMA Ksh 257.81 million.

This analysis shows that Holt-Winters methods tend to overestimate revenues, especially in Q2 and Q4, while SARIMA provided more conservative forecasts that are generally closer to actual figures but sometimes underestimate revenue.

The preferred forecasting model for the Domestic tax revenues in Kenya.

The SARIMA (0,1,1)(0,1,0)₁₂ model had superior predictive capabilities and was preferred for forecasting domestic tax revenues. It exhibited the lowest prediction errors across key metrics, MAE (4,998.15), MAPE (7.67%), and MASE (0.38), indicating superior accuracy compared to both the additive and multiplicative Holt-Winters models. Although all models achieved MAPE values below 10%, signifying excellent forecasts, SARIMA consistently had the smallest errors.

Further supporting this conclusion, SARIMA achieved the lowest RMSE (8,219.96) and MAE (6,432.64) values for the quarter 1 to quarter 4 summaries computed for each of the models compared to the actual amounts recorded in 2021, relative to the Holt-Winters methods, which showed higher error metrics and greater deviation from actual revenue values. While the multiplicative Holt-Winters model had a slightly better MAE than the additive version, it performed lower in RMSE, suggesting the seasonal pattern was not strongly multiplicative.

The Diebold-Mariano test showed a significant difference in predictive accuracy favoring SARIMA over the additive Holt-Winters model (DM = -3.499, $p < .05$), and a marginal but not statistically significant advantage over the multiplicative Holt-Winters model (DM = -1.972, $p = .074$). The Model Confidence Set (MCS) procedure further confirmed SARIMA as the only statistically superior model at the 5% significance level ($p = .027$), excluding both Holt-Winters variants as inferior.

In summary, while Holt-Winters models performed well, SARIMA was the most reliable and accurate model for forecasting domestic tax revenues in this study, particularly in capturing seasonalities and data trends effectively.

5.4 Limitations

This research project acknowledges certain limitations and assumptions. There exist many external factors that influence the economy of country at any given time that are not included in the model. These factors affect the revenue collections. These include: political unrest and demonstrations, terrorism activities, changes in government policies, global wars, level of taxpayer compliance, among others. Additionally, the COVID-19

pandemic, which was reported in Kenya from March 2020, had a huge impact on the nation's economy, and also the revenue collection dropped. These uncontrolled factors result in structural breaks, which potentially reduce the accuracy of generated models to forecast the revenues in Kenya.

5.5 Suggestions for Future Research

The structural breaks and discontinuities in the data distort the data trends, hence reducing the consistency and accuracy of data models. These need to be considered in financial data modelling due to their volatility. Use of Kalman filter and state space models greatly handles these effects within the data, hence accounting for the discontinuities, which results in more accurate modelling results. Additionally, incorporating the exogenous and macroeconomic variables in revenue forecasting further increases the accuracy levels, hence increasing the model's reliability. These model regressors accurately capture the externalities that could impact the economy.

Incorporating machine learning techniques and other innovative approaches in forecasting offers a great opportunity to improve prediction using complex data patterns. These technologies can be trained to capture behaviors and other external factors in the economy, which results in accurate data, hence increasing the accuracy of financial data models. These approaches are great opportunities for research in time series forecasting since they could solve the problem of external shocks, structural breaks, and complex data sets.

REFERENCES

- Abonazel, M. R., & Abd-Elftah, A. I. (2019). Forecasting Egyptian GDP using ARIMA models. *Reports on Economics and Finance*, 5(1), 35–47.
- Amadala, V. (2021, May 20). Treasury told to set logical budget to cut excess borrowing. *The Star*. <https://www.the-star.co.ke/business/2021-05-20-treasury-told-to-set-logical-budget-to-cut-excess-borrowing>
- Atoyebi, S. B., Olayiwola, M. F., Oladapo, J. O., & Oladapo, D. I. (2023). Forecasting Currency in Circulation in Nigeria Using Holt-Winters Exponential Smoothing Method. *South Asian Journal of Social Studies and Economics*, 20(1), 25–41. <https://doi.org/10.9734/sajsse/2023/v20i1689>
- Ayakeme, T. I., Biu, O. E., Enegesele, D., & Wonu, N. (2021). Forecasting of Bayelsa State Internally Generated Revenue using ARIMA Model and Winters Methods. *International Journal of Statistics and Applied Mathematics*, 6(1): 107–116
- Boonzaaier, W. (2012). *Revenue forecasting practices: Current international developments and the case of South Africa*. South African Revenue Service.
- Box, G. E., Jenkins, G. M., Reinsel, G. C., & Ljung, G. M. (2015). *Time series analysis: Forecasting and control* (pp. 18–23). John Wiley & Sons.
- Brew, L., & Wiah, E. N. (2012). An assessment of the efficiency in the collection of Value Added Tax revenue in Tarkwa-Nsuaem Municipality (Ghana) using time series model. *British Journal of Arts and Social Sciences*, 6(2), 140–150.
- Cetin, B., & Yavuz, I. (2020). Comparison of forecast accuracy of ATA and exponential smoothing. *Journal of Applied Statistics*, 48(13-15), 2580–2590.
- Chang, X., Gao, M., Wang, Y., & Hou, X. (2012). Seasonal autoregressive integrated moving average model for precipitation time series. *Journal of Mathematics and Statistics*, 8(4), 367–373.
- Chang, Y. W., & Liao, M. Y. (2010). A seasonal ARIMA model of tourism forecasting: The case of Taiwan. *Asia Pacific Journal of Tourism Research*, 15(2), 215–221.
- Chimilila, C. (2017). Forecasting tax revenue and its volatility in Tanzania. *African Journal of Economic Review*, 5(1), 84–109.
- Diebold, F. X., & Mariano, R. S. (2002). Comparing predictive accuracy. *Journal of Business & Economic Statistics*, 20(1), 134–144.
- Ergüven, M. H., Yılmaz, A., & Kutlu, D. (2015). Hybrid tourism within the context of touristic product diversification: Glamping. *Journal of Academic Social Science Studies*, (41), 255–265.
- Fattah, J., Ezzine, L., Aman, Z., El Moussami, H., & Lachhab, A. (2018). Forecasting of demand using ARIMA model. *International Journal of Engineering Business Management*, 10, Article1847979018808673.
- Fauzi, N. F., Ahmadi, N. S. ., Shafii, N. H. ., & Ab Halim, H. Z. . (2020). A Comparison Study on Fuzzy Time Series and Holt-Winter Model in Forecasting Tourist Arrival

- in Langkawi, Kedah. *Journal of Computing Research and Innovation*, 5(1), 36–45. <https://doi.org/10.24191/jcrinn.v5i1.138>
- Fisher, J. M. (2014). Fairer shores: Tax havens, tax avoidance, and corporate social responsibility. *Boston University Law Review*, 94, 337–384.
- Garrett, T. A., and Leatherman, J. C. (2000). *An Introduction to State and Local Public Finance*. West Virginia University, Regional Research Institute.
- Gathondu, E. K. (2014). *Modeling of wholesale prices for selected vegetables using time series models in Kenya* [Doctoral dissertation, University of Nairobi].
- Habibur Rahman, M., Salma, U., Moyazzem Hossain, M., & Tareq Ferdous Khan, M. (2016). Revenue Forecasting using Holt–Winters Exponential Smoothing. *Research & Reviews: Journal of Statistic*, 5(3), 19–25.
- Harelimana, J. B. (2018). The role of taxation on resilient economy and development of Rwanda. *Journal of Finance and Marketing*, 2(1), 28–39.
- Hasibuan, L. H., Musthofa, S., Putri, D. M., & Jannah, M. (2023). Comparison of seasonal time series forecasting using SARIMA and Holt-Winters exponential smoothing: Case study of West Sumatra export data. *BAREKENG: Journal of Mathematics and Its Applications*, 17(3), 1773–1784.
- Hidayat, R., Ilyas, M., & Yuliani. (2024). Multiple nonparametric regression approach for regional mapping by considering the significance of the independent variables. *IAENG International Journal of Applied Mathematics*, 54(3), 552–561.
- Holt, C. C. (1957). *Forecasting seasonals and trends by exponentially weighted averages* (O.N.R. Memorandum No. 52). Carnegie Institute of Technology.
- Hyndman, R. J., & Khandakar, Y. (2008). Automatic time series forecasting: The forecast package for R. *Journal of Statistical Software*, 27(3), 1–22.
- Hyndman, R., Koehler, A. B., Ord, J. K., & Snyder, R. D. (2008). Forecasting with exponential smoothing: The state space approach (pp. 230–250). Springer Science & Business Media
- ICPAK. (2016). *Kenya's revenue analysis 2010-2015: A historical perspective to revenue performance in Kenya* (pp. 1–4). Institute of Certified Public Accountants of Kenya.
- Jenkins, G. P., Kuo, C. Y., & Shukla, G. (2000). *Tax analysis and revenue forecasting* (pp. 7–14). Harvard Institute for International Development, Harvard University.
- Jochimsen, B., & Lehmann, R. (2017). On the political economy of national tax revenue forecasts: Evidence from OECD countries. *Public Choice*, 170(3-4), 211–230
- Kelkar, M., Borsa, C., & Kim, L. (2021). Time-Series Statistical Model for Forecasting Revenue and Risk Management. *Journal of Student Research*, 10(3).
- Kenya Institute for Public Policy Research and Analysis. (2016). *Kenya economic report 2016: Fiscal decentralization in support of devolution*. KIPRA. <http://repository.kippra.or.ke:8080/xmlui/handle/123456789/1916>

- Kenya Revenue Authority. (2019). *Annual revenue performance report FY 2018/19*. Kenya Revenue Authority. <https://www.kra.go.ke/images/publications/Revenue-Performance-Report-2018-19.pdf>
- Kithure, M. E. (2018). Effectiveness of time series models on forecasting of Value Added Tax revenue in Kenya Revenue Authority (pp. 1–11). Kenya School of Revenue Administration / Jomo Kenyatta University of Agriculture and Technology
- Koirala, T. P. (2013). Government revenue forecasting in Nepal. *NRB Economic Review*, 24(2), 47–60. <https://doi.org/10.3126/nrber.v24i2.52727>
- Kyobe, A. J., & Danninger, S. (2005). *Revenue forecasting—How is it done? Results from a survey of low-income countries* (IMF Working Paper No. 05/24). International Monetary Fund. <https://www.imf.org/external/pubs/ft/wp/2005/wp0524.pdf>
- Larmore, E. A. (2016). *Methodologies for forecast modeling for small areas with limited data availability and unique tax structures* (pp. 51–52). University of Nevada, Reno.
- Lilian, O. (2015). Kenya loses over Sh600bn every year in tax evasion.
- Mahsin, M. D., Akhter, Y., & Begum, M. (2012). Modeling rainfall in Dhaka division of Bangladesh using time series analysis. *Journal of Mathematical Modelling and Application*, 1(5), 67–73.
- Makananisa, M. P. (2015). *Forecasting annual tax revenue of the South African taxes using time series Holt-Winters and ARIMA/SARIMA models* (Master's thesis, University of South Africa). University of South Africa Institutional Repository.
- Makridakis, S., Wheelwright, S. C., & Hyndman, R. J. (2008). *Forecasting methods and applications* (3rd ed.). John Wiley & Sons.
- Mohammed, F. A., & Mousa, M. A. (2020). Applying Diebold–Mariano test. In *Theory and applications of time series analysis: Selected contributions from ITISE 2019* (p. 443). Springer.
- Moyi, E., & Ronge, E. (2006). *Taxation and tax modernization in Kenya: A diagnosis of performance and options for further reform* (pp. 1–12). Institute of Economic Affairs.
- Munarsih, E., & Saluza, I. (2020). Comparison of exponential smoothing method and autoregressive integrated moving average (ARIMA) method in predicting dengue fever cases in the city of Palembang. *Journal of Physics: Conference Series*, 1521(3), 2–9. <https://doi.org/10.1088/1742-6596/1521/3/032100>
- Ofori, M. S., Fumey, A., & Nketiah-Amponsah, E. (2020). Forecasting value added tax revenue in Ghana. *Journal of Economics and Financial Analysis*, 4(2), 63–99
- Otu, O. A., Osuji, G. A., Opara, J., Mbachu, H. I., & Iheagwara, A. I. (2014). Application of SARIMA models in modelling and forecasting Nigeria's inflation rates. *American Journal of Applied Mathematics and Statistics*, 2(1), 16–28.
- Pongdatu, G. A. N., & Putra, Y. H. (2018). Seasonal time series forecasting using SARIMA and Holt Winter's exponential smoothing. *IOP Conference Series: Materials Science and Engineering*, 407(1), 012153.

- Poulson, B. W., & Kaplan, J. G. (2008). State income taxes and economic growth. *Cato Journal*, 28, 53–75.
- Rahman, M. H., Salma, U., Hossain, M. M., & Khan, M. T. F. (2016). Revenue forecasting using Holt–Winters exponential smoothing. *Research & Reviews: Journal of Statistics*, 5(3), 19–25.
- Saayman, A., & Botha, I. (2017). Non-linear models for tourism demand forecasting. *Tourism Economics*, 23(3), 594–613.
- Saayman, A., & Saayman, M. (2008). Determinants of inbound tourism to South Africa. *Tourism Economics*, 14(1), 81–96.
- Samuel, F. K., & Kibua, T. K. (2019). Seasonal autoregressive integrated moving average model for tax revenue forecast in Kenya. *European International Journal of Science and Technology*, 8(6), 11–30.
- Signé, L. (2016). *How to implement domestic resource mobilization (DRM) successfully for effective delivery of sustainable development goals (SDGs) in Africa: Illustrative actionable solutions for policy leaders.*
- Susan, W. G., Anthony, G. W., & John, M. K. (2015). Forecasting inflation rate in Kenya using SARIMA model. *American Journal of Theoretical and Applied Statistics*, 4(1), 15–18.
- Suwanvijit, W. (2014). Forecasting tourist arrivals in IMT-GT using Lee-Carter method. In *AFBE 2014 Conference Papers* (p. 240). Thaksin University.
- Tasi’u, M., Usman, A. M., & Garba, H. D. (2024). Modelling and Forecasting Nigeria’s Tax Revenue: A Comparative Analysis of SARIMA and Holt-Winters Models. *UMYU Scientifica*, 3(3), 118–129. <https://doi.org/10.56919/usc.2433.014>
- The National Treasury and Economic Planning. (2025). *Budget highlights: The Mwananchi guide for the FY 2025/26 budget.* Government of Kenya.