THE TWO-DIMENSIONAL NON-HOMOGENEOUS POISSON PROCESS BASED ON EXTREME VALUE THEORY FOR VALUE AT RISK

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DECLARATION

This research project is my original work and has not been presented elsewhere for a degree or any other award.

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DEDICATION

I dedicate this research to my dearest family, starting with my lovely wife, Doris Ndinda Mbuu, and my daughter, Leslie Wayua Mbuu; my dearest father, Mr. Francis Matuku Mbatha; and my dearest Mother, Justine Wayua Matuku, for their love, support, and encouragement throughout my research work. Their support, both moral and financial, has been a pillar of strength in my academic journey. To my siblings, Moses Kioko, Elizabeth Nditi, Vincent Mbatha, Mariane Twili, and Veronica Wandii, who have been there to support me and been a source of inspiration and joy, thank you for always being there with words of encouragement and support. Lastly, I dedicate this work to my future self, as a reminder of the perseverance, resilience, and determination that it took to achieve this milestone.

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LIST OF ACRONYMS

2D- NHPP	Two-Dimensional Non-Homogeneous Poisson Process
ACF	Autocorrelation function
ARCH	Autoregressive Conditional Heteroscedasticity
ARIMA	Autoregressive Integrated Moving Average
CBK	Central Bank of Kenya
ES	Expected Shortfall
EVT	Extreme Value Theory
GARCH	Generalized Autoregressive Conditional Heteroscedasticity
GEV	Generalized Extreme Value
GPD	Generalized Pareto Distribution
MRL	Mean Residual Life
NSE	Nairobi Securities Exchange
PACF	Partial Autocorrelation Function
VaR	Value at Risk

LIST OF SYMBOLS

P_t	price of the NSE 20 share index at time t
P_{t-1}	price of the NSE 20 share index at time t-1
r_t	return of the NSE 20 share index at time t
α	alpha, for scale parameter
β	beta, for location parameter
K	shape parameter
T	Total number of observations
D	baseline interval, set to 252 trading days
P	level of confidence
γ	gamma, for shape parameter coefficients
θ	theta, for location parameter coefficients
δ	delta, for scale parameter coefficients
η	eta, for threshold
Λ	lambda, for rate parameter for the Poisson distribution
σ_t^2	is the square of volatility at time t
ε_t	is the error term with student t distribution
y_t	the mean corrected return at time t

DEFINITION OF TERMS

An **Extreme value**: a data point in a dataset that is significantly larger or smaller than the majority of observed values

A **parameter*:*** is a characteristic of a population

A **coefficient**: a numerical value that quantifies the relationship between an independent variable and the dependent variable, indicating the change in the dependent variable for a unit change in the independent variable

Non-homogeneous: refers to a statistical framework where the parameters governing extreme events are allowed to vary over time or in response to external covariates, rather than remaining constant

Poisson process: stochastic process that models the occurrence of events randomly over time or space

ABSTRACT

Estimating Value at Risk (VaR) is an important aspect in management and mitigation of risk for institutions, individual investors, and markets. Extreme events have in the past caused significant financial losses, sometimes leading to market crashes that have had a profoundly negative impact on many investors, companies, institutions, and governments. Hence, there is a need to accurately model and predict extreme events. The Extreme Value Theory which applies the Generalized Extreme Value Distribution and the Generalized Pareto Distribution offers a robust framework for modeling extreme events and tail risks. This study focused on the peak over the threshold method, where a two-dimensional non-homogeneous Poisson process model based on extreme value theory was employed, and the model's parameters are linear functions of the interest rates and volatility. Volatility and interest rates have had a significant impact on investment returns and financial losses. The Nairobi Securities Exchange 20-share index and Central Bank of Kenya interest rates datasets were used, with daily observations spanning 10 years from January 2, 2014, to December 31, 2023. The maximum likelihood and optimization methods were employed to estimate the coefficients of the linear relationships between interest rates and volatility, as well as the location, scale, and shape parameters. The volatility variable was found to be positively related to all three model parameters, shape, scale, and location while interest rates were negatively related. Unlike the traditional model that assume risks are static, this approach assumes varying market risks and conditions with time. It becomes a reliable tool for markets experiencing frequent changes, which allows them to collect accurate data which would be challenging if the traditional method was used. Future studies should explore the non-linear associations and also other explanatory variables.

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CHAPTER ONE

INTRODUCTION

1.1 Background of the Study

Investments and portfolio involve risks which must be accurately estimated to ensure prevention and effective management in case they occur. Value at Risk (VaR), described as the highest loss that an investment could incur within a particular timeline and under usual market environment. Through it, entities predict risks that would reduce asset or portfolio value in a certain time (Žugec, 2019). Through the VaR, a monetary value of the amount that would be lost from the initial investment due to risks is determined (Macieira, 2021). For instance, when the VaR provides a $1 million figure with a 95% confidence in a day, it means that the probability of loss is 5% and if the market remains as expected and the loss occurs, more than $1 million in value would be lost. Different tools including simulation, Monte Carlo and parametric are useful in calculating VaR (Mukhodobwane, 2022). Selecting the method to use depends on various factors such as accuracy needed available data and the complexity of the investment. Notably, the methods have benefits and weaknesses which must be considered to ensure accurate risk determination. In some instances, it would be better to use more than one tool especially when they complement each other and limit chances of inaccuracies.

Different institutions including financial entities, firms and regulatory organizations use VaR. Besides helping them to manage possible risks, the tool is helpful in making financial decisions, adhere to laws and determine risks limits. However, it is important to note that the tool only assesses risks when the market environment is normal and within a specific timeline. In case of adverse events which are infrequent and directly affect investments, additional strategies must be used (Nemukula & Sigauke, 2021). Considering that external factors affect portfolio, there is a need to include other risk management tools which provide a comprehensive report which aids in making informed decisions. Regular VaR tools assume that market conditions remain normal and they are independent (Boano-Danquah et al., 2020). It causes inconsistences due to the lack of considering tail risks yet most financial markets handling critical transactions such as returns involve high peaks and tail risks. As a result, an urgency to develop more reliable and sophisticated methodologies to ensure accurate risks assessments where extreme outcomes and tail risks are considered is increasing. One of the current innovative frameworks is the Extreme Value Theory (EVT), which collects detailed and timely data, hence helps to identify tail risks enabling making of informed decisions (Salem et al., 2022). EVT focusses on monetary behaviors involving extreme events and provides accurate reports making it more reliable than the VaR. Nevertheless, relying on the traditional EVT strategies could cause biases because they rely on complex dependence structures. Thus, organizations utilizing EVT to manage risks should use advanced tools.

Recently, the need to integrate dependence structures in VaR frameworks have developed. A reliable alternative is the Two-Dimensional Non-Homogenous Poisson Process (2D-NHPP) because it is flexible in identifying dependences in different time series. Using the two-dimensional space, the model calculates possibilities for extreme events, determining marginal distributions and dependence element of the financial data (Fabiani et al., 2021). The primary aim of this study was to narrow the gap between the 2D-NHPP and the EVT by integrating the VaR approach. As (Beaumard, 2023) asserts, combining the strengths of these frameworks expands the capacities of the VaR and accuracy. As a result, entities' capacities to manage extreme events and tail risks with complicated dependence elements increase.

Globalization and technological advancement has increased dependability between the financial markets. This has heightened the importance of more reliable risk management methodologies. The traditional strategies are not comprehensive because they do not consider the entire systemic risks leading to severe losses and other negative outcomes when markets experience challenges. By combining the key elements of EVT and the 2D-NHPP, this study becomes a suitable avenue to resolve issues caused by systemic risk by enabling a detailed analysis of dependencies and effects in diverse financial markets. Additionally, this framework has benefits beyond risks management. It gives valuable insights on laws and regulations, pricing investments and optimization which also plays a role in making informed decisions. When estimations about tail risks and extreme events are accurate, entities in the financial market not only make informed decisions but they also take mitigation measures which build resilience when market shocks occur. Accurate data also benefits policy makers because they pass regulations which strengthen market stability and implement measures which limit chances of systemic crisis (Cai & Daskalakis, 2011).Therefore, this study has a high generalizability and the different institutions can utilize the framework to ensure positive outcomes.

1.2 Statement of the Problem

Investors purchasing shares in companies worry about uncertainty of market dynamics and predictability of future prices (Salem et al., 2022).. This framework targets them to ensure they feel confident in their investments. The advanced statistical techniques will enable them to calculate estimated losses within a specific time and based on a particular confidence level. Specifically, a more accurate VaR will improve risks determination and management addressing the challenges and uncertainties that investors are currently experiencing. Thus, through the Extreme Value Theory (EVT) a more reliable framework that examine extreme events and tail risks is developed, which is an improvement of the traditional methodologies. The regular methodologies have weaknesses that increase possibilities for biases and inaccurate reports which has increased the need for more developed interventions which are multidimensional. The two-dimensional non-homogenous Poisson process is a reliable alternative which captures extreme events, considering the dynamics in the financial markets. It incorporates explanatory variables including interest rates and volatility therefore improving the capacity of the VaR to accurately estimate investment risks.

1.3 Objectives

1.3.1 General objective

The primary objective is to apply the two-dimensional non-homogenous Poisson process based on the extreme value theory model to estimate value at risk.

1.3.2 Specific objectives

I. To analyze the statistical properties of NSE 20 share index return series.
II. To determine the parameters of two-dimensional non-homogeneous Poisson process model based the extreme value theory.

III. To estimate the value at risk for investment in the NSE 20 share index.

1.4 Research hypothesis

I. $H_{01:}$ NSE 20 share index returns data is not appropriate for the utilization of Extreme Value Theory

II. $H_{02:}$ The EVT model based on the two-dimensional non-homogeneous Poisson process does not accurately capture extreme events and tail risks in financial markets

III. $H_{03:}$ VaR estimate model parameters are not sensitive to different market conditions and scenarios

1.5 Justification of the Study

The study is essential considering the increased sophistication and uncertainty in the financial markets, yet the traditional value at risk and EVT are unreliable. Through the two-dimensional non-homogenous Poisson process, the risk management process will advance highly benefiting the financial market because they improve VaR accuracy. The regular VaR does not assess extreme outcomes and tail which increases biases and risks which makes the reports inaccurate. Combining the two-dimensional non-homogenous Poisson process and the EVT improves VaR accuracy since market changes in interests' rates and returns volatility are assessed. These explanatory valuables empower institutions to combat preventable losses through making informed decisions. Significance of the Study

This study strengthens the VaR framework to improve risks management for financial institutions and the general market. Empowering the entities to collect and analyze accurate data improves comprehension and management of adverse risks. This study helps to mitigate the adverse impacts of market volatility and enhance the stability and resilience of financial systems. Moreover, the insights gained from this study can inform regulatory bodies and policy makers in developing more strategic and effective risk management standards for the financial industry. The inclusion of explanatory variables of interest rates and volatility underscores how the economic policy helps to mitigate the effect of extreme events. The positive relationship between the calculated volatility of returns and parameters (shape, location, scale) of the EVT distribution helps inform risk managers about how periods of high volatility increase baseline risk, increase the likelihood of occurrence, and magnify the magnitude of extreme returns. This provided regulators and risk managers with better insight to devise more effective risk management strategies. The negative relationship between interest rates also provided important insights into how the government mitigates risks and prevents market crashes by setting more favorable interest rates.

1.7 Scope of the Study

The study centered on the performance of NSE 20 share index as the primary dataset, which represents of the 20 highest-performing companies listed on the NSE. The data covered daily returns for 10 years, from January 2, 2014, to December 31, 2023, capturing both normal and extreme market events. The study employed the extreme value theory and a two-dimensional non-homogeneous passion process as methodologies for data analysis and estimation of value at risk. The study also investigated the suitability of returns for applying extreme value methods by analyzing the distributional properties of the NSE 20 share index. The primary outcome was the calculation of Value at Risk (VaR) using a two-dimensional non-homogeneous Poisson process, based on extreme value theory, which captures extreme market events.

1.8 Assumptions of the Study

The research made the assumptions that:

1. The NSE 20 share index is sufficiently liquid which ensures that the prices and return data used are reliable and reflect the true market behavior.
2. The return data series is assumed to be stationary implying that both the mean and variance remain constant over time which is essential for applying the EVT models.
3. The return series exhibited volatility clustering, that is, periods of high volatility are followed by periods of low volatility.
4. Return data contains sufficient extreme events (tail events) to accurately apply EVT. If extreme losses or gains are not adequately represented in the dataset, EVT may not capture the true risk.

5. Extreme market events do not occur at a constant rate but vary over time. This is the rationale for using a non-homogeneous Poisson process, which enables the intensity of extreme events to vary according to market conditions.
6. The occurrence of extreme events follows a Poisson distribution suggesting that the probability of an extreme event occurring in any given period depends on the event rate, which can vary in a non-homogeneous manner.

CHAPTER TWO

LITERATURE REVIEW

2.1 Theoretical Review

2.1.1 Value at Risk

Value at Risk (VaR), described as the highest potential loss that an investment could incur within a particular timeline and under normal market conditions. Through it, entities predict risks that would reduce asset or portfolio value in a certain time. It was first used in the risk metrics approach that J.P Morgan developed. Since its development, the tool has been improved, more methods of its measurement developed and become popular in the financial sector. Today, diverse calculation methods such as simulation and the Monte Carlo approach are used (Jorion, 1996). The regular VaR methods are used to assess market risks but they do not consider adverse events and market instabilities, hence they do not provide comprehensive reports. Their weaknesses were evident during the 2008 financial crisis since data on tail risks and fat tails were not collected which delayed the resolving process. It is important to close this gap by utilizing advanced methods especially the EVT which consider tail implications.

2.1.2 Extreme Value Theory

EVT has instrumental statistical tools which evaluates possibilities for adverse events which are not within the expected occurrences. Its core element is the generalized extreme value (GEV) that combine Weibull, Fréchet and Gumbel which are advanced elements that assess tail behaviors of a distribution.

EVT is reliable in estimating tail risks making it useful in financial risk management because it assess possibilities for adverse financial losses. Compared to the traditional VaR methods, EVT better predicts risks of extreme losses that could occur when sudden shocks occur (Weng et al., 2018). However, the threshold levels must be carefully selected and the data collected must be high quality to guarantee that the EVT would effectively capture the tail risks. Additionally, the model relies on one-dimension approach and does not consider the interdependence between time and volatility indicating that it is not very efficient for systems that regularly change and those that are complicated. This demonstrates the need to integrate two-dimensional space models such as the Poisson process and extreme value theory to narrow the gap.

2.1.3 Non-Homogeneous Poisson Process

The non-homogenous Poisson process is a vital model that explores random events occurring in a specific time. Fields such as insurance mathematics, telecommunication and financial institutions highly utilize it (Marquez et al., 2009). As the name suggests, (homogenous), this model assumes events occur uniformly and independently within a time and at a constant average where the symbol λ (lambda) is used. This indicates that the frequency of occurrences in a specific time adheres to the Poisson distribution while the inter-arrival times in between the events adhere to an exponentially probability pattern.

Majority of the real-world systems in the financial market do not follow a perfect pattern due to structural breaks, external factors and high volatility. The non-homogenous Poisson process (NHPP) combats these weaknesses by integrating the intensity function λ (t) which adjusts with time and market dynamics. The dynamic element enables the NHPP to become a valuable tool for evaluating changes in events occurrences including volatility spikes, shifts due to policy implementations and market stresses. Notably, the NHPP is a generalization version of the usual Poisson strategy, whereby the intensity of the event depends on time (Marquez et al. 2009). It becomes applicable for markets where the risks changes with time and they do not follow a defined pattern. It is popular in insurances but the financial field has also recently adopted it. The quick acceptability is because extreme events cause significant losses which the use of this model limits.

Current studies prove that NHPP is not only effective in theory but also in real life applications and especially in financial time series data where market conditions change with time. Badescu et al., (2005) claimed that the model has helped in timing market crashes and seasons when volatility is high since they are non-homogenous. By determining changes in the market, NHPP allows risks estimations enabling the entities make necessary changes to avoid the adversities. Therefore, integrating the non-homogenous Poisson process with the extreme value is a reliable theoretical foundation for utilizing the two-dimensional Poisson model that is implemented in this study.

2.2 Empirical Review

2.2.1 Statistical Properties of the Returns Data Series Using Extreme Value Theorem.

Risk management approaches such as VaR and Expected Shortfall (ES) use different quantile assessment approaches. Most financial returns have characteristics that are non-normal including skewness, much tails and kurtosis (Miniussi et al., 2020). Since they have more tails than the regular distribution, the likelihood of adverse values are high which justifies the need to implement the extreme value theory in this research study. Alam et al., (2018) assets that "the extent to which extreme losses are projected to happen with little likelihood are denoted by these quantiles." Through the EVT, risks in financial markets are determined by pointing out adverse events via systemic parameter prediction.

Implementing EVTs in financial institutions indicate the linkage between theory and practice. The theoretical formulations are practically applied which guarantees predictability (Li et al., 2022). Quality pricing methodologies are vital in the finance field during investments optimization, pricing and managing risks. With the changing dynamics in the financial market, the extreme-value theorems become helpful by laying a theoretical foundation for the usage of an optimal multidimensional approach as evident in this study. This is attained by considering both the maximum and minimum values in the models. As a result, the usage of EVTs gets easier when developing risks management techniques, pricing as well as valuation (Weng et al., 2018).

Entities such as research and telecommunications use Poisson elements for simulations after events that occur from time to time occur. The processes are mainly applicable for the events that occur over a long time (Jiang, et al., 2023). Examples of such events include fast increase of asset value and in making insurance portfolio claims. The event rate increases through the NHPP and the non-homogenous Poisson Processes which results in prolonging the Poisson process (Żuławiński et al., 2023). It is mostly suitable for simulations which are time varying, like the variable rates.

2.2.2 EVT Approach Based on the Two-Dimensional Non-Homogeneous Poisson Process

The extreme value theory helps in managing extreme events because they are broader and comprehensive (Hofert et al., 2020). Drastic changes in assets prices and economic collapses are examples of adverse effects where EVT are helpful. Combining EVT with two dimensional non-homogenous Poisson processes help in understanding the events and tail risks which improves the preparedness of the market in handling them. They also consider fluctuations experienced with time because the non-homogenous Poisson processes are highly flexible (Blier-Wong, 2023). The processes are highly useful for simulations of events that randomly occur over a period of time (Tepetepe et al., 2022).

In this study, the occurrence of extreme events were assumed to be random. According to Cornwell et al. (2023), it is characterized by independent increases and a steady average rate of event occurrence. The two-dimensional non-homogeneous Poisson process allows for a dynamic and flexible modeling of the occurrence rates of extreme events. EVT offers robust statistical techniques for modelling the severe event-prone regions near the distribution's tails. The model's ability to capture deep interconnections and changes in extreme event occurrences stems from its consideration of two key dimensions: time and asset type (Frey et al., 2020). It aids in developing effective risk mitigation measures and provides a more thorough grasp of high threats.

According to Choudhury & Daly (2019), a potent framework for precisely capturing extreme events and tail risks in financial markets is offered by the combination of EVT and two-dimensional non-homogeneous Poisson processes. This strategy combines the best features of the two approaches, providing strong statistical tools for tail risk assessment and flexibility in modelling different event rates. Implementing this model in reality enhances risk management procedures and provides a deeper understanding of how extreme financial events behave (Halkos & Tsirivis, 2019).

VaR calculations derived from historical data may not account for changing market conditions; therefore, the use of non-homogeneous models will capture these changing conditions. VaR estimations have a confidence level associated with them hence different levels of confidence (e.g., 95%, 99%) provide different levels of the

Estimate. VaR estimates increase with higher confidence levels, although more conservative estimates are provided at lower confidence levels. However, extreme events may not be fully captured (Adams et al., 2020).Hence, the choice of the confidence level presents both a financial problem and a statistical problem. This study provided estimates that are less sensitive due to the integration of the non-homogeneous Poisson process to capture the occurrence of events and the EVT to model the magnitude of such events.

2.2.3 Sensitivity of VaR Estimates

Traditional VaR estimations, however, depend on several model assumptions and the distribution of returns (Cinelli & Hazlett, 2020). A breakdown of the ways in which these factors could influence the estimation of VaR. According to Afonso et al. (2018), VaR calculations derived from a normal distribution presume a Gaussian distribution for returns. On the other hand, non-normal features like skewness and kurtosis are frequently observed in financial returns. Therefore, the assumption of the underlying distribution is a key factor and the VaR estimates are sensitive to the distribution assumed. Therefore, this study was not be based on an assumption that the returns followed a specific distribution to provide more accurate results.

The distribution of returns exhibits skewness or fat tails that deviate from the normal distribution; therefore, assuming normal distribution in the returns data series when estimating Value at Risk (VaR), could lead to a potential underestimation of tail risk. VaR estimations in this study were enhanced by utilizing various distributional assumptions, including the Generalized Pareto Distribution (GPD), which was extended to the 2D-NHPP model. VaR estimates differ depending on how frequently historical data is used (daily, monthly, yearly) and the length of the data series. While prolonged time frames may smooth out extreme events (Antonakakis et al., 2020), shorter periods or higher-frequency datasets may result in more volatile VaR estimations. Over time, financial markets show non-stationary behavior hence, the daily returns data will be used to provide more accurate and reliable VaR estimates in this research.

The tail behavior of financial returns that places more emphasis on extreme occurrences than on the overall tendency is provided by Extreme Value Theory (EVT) (Majumdar et al., 2020). Understanding and managing the risks associated with uncommon but significant events, such as market collapses or significant price swings, is made easier using this method.

CHAPTER THREE

METHODOLOGY

3.1 Research Design

Quantitative retrospective research design approach was utilized in this study, using financial market data from the Nairobi Securities Exchange (NSE) 20-share index, which tracks the 20 highest-performing companies. Data was obtained from the NSE offices. R statistical software was used to carry out the analysis. The initial analysis consisted of time series plots and descriptive statistics, as well as tests for normality, serial correlation, stationarity, and the presence of ARCH effects, to understand the distributional properties of returns and their suitability for applying extreme value theory. The study incorporated the volatility of returns and the central bank interest rates as explanatory variables. These explanatory variables were incorporated as linear factors of shape, location, scale parameters of the model and data used to estimate the coefficients associated with the explanatory variables of the two-dimensional non-homogeneous parameters. The observed interest rates and calculated volatility for the same period were used to estimate the location, shape, and scale parameters, which were then used to calculate VAR estimates at various confidence levels.

3.2 Sources of Data

The data was sourced from CBK website and NSE offices and included the NSE 20 share index and CBK interest rates, with daily observations spanning 10 years from 02/01/2014, to 31/12/2023. The choice to use daily data provided a granular view of market movements, capturing short-term fluctuations and extreme values, which are essential for accurate Value at Risk (VaR) calculations. A census approach was employed in this study, utilizing all observations collected over the chosen period for analysis.

3.3 Data transformation

During the analysis, the log daily returns of the NSE 20 share index were used. The conversion to log returns was made due to the properties of log returns compared to returns or raw prices data listed below:

Time additivity; unlike simple returns, log returns can be summed across periods to calculate cumulative returns, which simplifies multi-period return calculations and portfolio performance assessments.

Log returns also tend to exhibit more stable statistical properties, such as approximate normality, making them suitable for use in econometric models that assume normally distributed variables, including linear regressions and volatility models like the GARCH model.

Another benefit of using log returns is their symmetry around zero, which ensures that gains and losses of the same magnitude have a more balanced impact on return measurements. This is particularly important in avoiding biases in risk and performance evaluations.

Log returns align with the concept of continuous compounding, a standard assumption in many theoretical finance models for instance the Black-Scholes model for pricing of options. This compatibility allows for more accurate and theoretically consistent modeling.

Log returns are also scalable and comparable across different assets or time frames, enhancing their usefulness in analyzing and comparing financial instruments with varying price levels or volatilities

The formula defines the log returns,

$$r_t = \ln \frac{p_t}{p_{t-1}}$$

Where;

r_t is the log returns at time t

p_t is the value of the NSE 20 share index at the time t

p_{t-1} is the value of the NSE 20 share index at the time $t-1$

3.4 Distributional properties of returns

Returns data series in finance often exhibit certain distributional properties that are important for risk management, portfolio optimization, and financial modeling. The key distributional properties of returns data series were tested to assess the suitability of the NSE returns data series for application of the EVT. This formed the basis for applying the EVT to the data of the share index return series. The properties and stylized characteristics of the returns data listed below were tested and analyzed.

Table 1: Diagnostic Statistics of Returns

Tests	Tests to be Used	Expected Conclusion
Normality	Q-Q plot and Shapiro Wilk test	The data is not normally distributed
Stationarity test	Augmented Dicker-Fuller (ADF) test	Returns data series is stationary
Serial correlation test	Autocorrelation function (ACF) and partial Autocorrelation function(PACF)	Short term serial correlation
Heteroscedasticity (presence or ARCH effects)	Chi-square test	Presence of ARCH effects and volatility clustering
Testing for Presence of Extreme Events	Time series plots	The returns data series contains extreme values

These distributional properties of returns data series play an important role in financial analysis, management and mitigation of risk, and portfolio optimization. Understanding and modelling these properties accurately is crucial for making well-informed investment choices and effectively mitigating financial risk.

3.5 Value at Risk

For an investment in a given number of shares, VaR can be described as the highest potential loss of a financial assets held over a specified period with a predetermined probability of occurrence. Suppose that at, at a specific point in time, the investor seeks to measure the risk associated with a long financial position taken for the next n periods and we define the change in the value of the asset from time t to t+n be as $\Delta V(n)$.Therefore, the value at risk over the given time interval $t \text{ to } t+n$ with probability p,

$$p = \Pr(\Delta V(n) \leq \text{VaR}) \qquad (1)$$

3.6 Extreme Value Theory

The return of a given investment calculated on a predetermined time interval, for example, daily or monthly returns is denoted by r_t. Such that the return series is denoted by $\{r_t\}$, for $t = 1,2, \dots, n$. In this study, the focus was on the minimum of the return series, which is of interest to extreme value theory and value-at-risk estimation for a long financial position. The EVT is based on finding two sequences, $\{\beta_n\}$ and $\{\alpha_n\}$, where $\alpha_n > 0$, such that the distribution of the normalized minimums $r_{(1)} = \frac{(r_{(1)} - \beta_n)}{\alpha_n}$ converges to a nondegenerate distribution as n goes to infinity. Where $\{\beta_n\}$ is the location series and $\{\alpha_n\}$ is a series of scaling factors.

The limiting distribution of the normalized minimum under the independence assumption is given below,

$$F(x) = \begin{Bmatrix} 1 - \exp(-(1 + kx)^{\left(\frac{1}{k}\right)} & \text{if } k \neq 0 \\ 1 - \exp\ (-\exp\ (x)\ ) & \text{if } k = 0 \end{Bmatrix} \qquad (2)$$

This distribution is known as the GEV distribution, and the parameter k represents the shape parameter of the distribution.

The GEV distribution is comprised of the three types of distributions which are dependent on the value of k, which is the shape parameter of the distribution

Type I: When $k = 0$, we have the Gumbel family. The C.D.F. is given by

$$F(x) = 1 - \exp\ (-\exp\ (x)\ )\ , -\infty < x < \infty, \qquad (3)$$

and represents the exponential like distributions

Type II: When $k < 0$, we have the Fréchet family. The C.D.F. is given by

$$F(x) = \begin{Bmatrix} 1 - \exp(-(1 + kx)^{\left(\frac{1}{k}\right)} & \text{if } x < -\frac{1}{k} \\ 1 & \text{otherwise} \end{Bmatrix}, \qquad (4)$$

and represents the heavy tailed distributions

Type III: When $k > 0$, we have the Weibull family. The C.D.F. is given by,

$$F(x) = \begin{Bmatrix} 1 - \exp(-(1 + kx)^{\left(\frac{1}{k}\right)} & \text{if } x > -\frac{1}{k} \\ 0 & \text{otherwise} \end{Bmatrix}, \tag{5}$$

and represents the light tailed distributions.

The MLE method was used to estimate the parameters β_n, α_n and k_n, using the extreme value distribution. Hence, the value at risk for a long position using this approach is given by,

$$\text{VaR} = \begin{Bmatrix} \beta_n - \frac{\alpha_n}{k_n}\{1 - [-n\ln(1-p)]^{k_n}\} \text{ if } k_n \neq 0 \\ \beta_n + \alpha_n \ln[-n\ln\ (1-p)] \quad \text{if } k_n = 0 \end{Bmatrix} \tag{6}$$

Where n is the sub-period and p is the chosen probability and,

β_n Location parameter

α_n scale parameter

k_n shape parameter

3.7 The Generalized Pareto Distribution

A threshold η was calculated using the quantile method and the mean residual life plot method for the return series, such that our interest was in returns that exceed the set threshold and by how much these returns exceed the set threshold. Let the value, $r_t - \eta$ be the amount of exceedance, observed at time t; therefore, the two-dimensional Poisson process focuses on the amount of exceedance and the particular point in when this exceedance occurs. The paired data $(t_{i,} r_t - \eta)$ now becomes the time of the occurrence of the exceedance and the amount of exceedance at that particular point in time. The laws of probability govern the probability of exceeding the given threshold; hence, the occurrence of exceedances follows the Poisson process. The exceedance are, $r_t - \eta = x$ such that $r_t = x + \eta$ given that the value of the return at the time t, exceeds the threshold. The distribution for a short position was applied to facilitate the derivation of the distribution of $r_t = x + \eta$, given that $r_t > \eta$, which aligns with the properties of the G.P.D. since losses for a short position are incurred when prices increase sharply. Therefore,

$$p(r \leq x + \eta \ /\ r > \eta) = \frac{p(\eta \leq r \leq x + \eta)}{p(r > \eta)} \tag{7}$$

Using the extreme value distribution, the conditional pdf becomes,

$$p(r \leq x+\eta \,/\, r>\eta) \approx 1-\left(1-\frac{kx}{\alpha-k(\eta-\beta)}\right)^{\frac{1}{k}} \qquad (8)$$

Where, $x > 0$ and$1-\frac{k(\eta-\beta)}{\alpha} > 0$,

as k approaches zero, i.e. $k \to 0$, the conditional distribution becomes,

$$p(r \leq x+\eta \,/\, r>\eta) \approx 1-\exp\left(-\frac{x}{\alpha}\right) \qquad (9)$$

The cumulative function for the distribution is given,

$$H_{k,\psi(\eta)}(x) = \begin{Bmatrix} 1-\left(1-\frac{kx}{\psi(\eta)}\right)^{\frac{1}{k}} \text{ for } k \neq 0 \\ 1-\exp\left(-\frac{x}{\psi(\eta)}\right) \text{ for } k=0 \end{Bmatrix} \qquad (10)$$

Where $\psi(\eta) = \alpha - k(\eta-\beta) > 0$

$x \geq 0$ when $k \leq 0$

$0 \leq x \leq \frac{\psi(\eta)}{k}$ when $K > 0$

This gives the pdf of the generalized Pareto distribution with parameters k and $\psi(\eta)$. Hence, the conditional distribution of r given $r > \eta$ is the G.P.D. with parameters k and $\psi(\eta)$ where $\psi(\eta) = \alpha - k(\eta-\beta)$

For $k = 0$, the distribution reduces to the exponential distribution, which does not exhibit the heavy tail characteristic of return data.

3.8 The two-dimensional non-homogenous Poisson process

This process is an extension of the classical Poisson process to two dimensions, where events occur randomly in both space and time. A Poisson process is a stochastic process that models the number of events, such as, arrivals, failures, or occurrences in continuous time. In a nonhomogeneous Poisson process, rate of occurrence of the random events varies with time or space. It is not constant. Consider a region in two-dimensional space e.g., a square, a circle, or any bounded area. Events or points occur randomly within this region and the intensity function (rate) varies across the region.

For each point (x, y), the intensity is given by $\lambda(x, y)$. The number of events in any sub region follows a Poisson distribution with an intensity proportional to the area of that sub region.

3.8.1 Mathematical Formulation:

The probability of observing k events in a sub region A is given by:

$$P\left(N(A)=k\right)=\frac{\left(\lambda(A)\right)^k e^{-\lambda(A)}}{k!} \tag{11}$$

Where, λ (A) represents the integral of the intensity function over the sub region A.

We extend the concept to accommodate the methods used in this study. We set a baseline time interval D to be a year, which contains 252 trading days. Let $t = 1,2,3,\ldots,T$ where T is the total number of observations. For a specified threshold η, exceedance times over the threshold are given by, $\{t_i, i = 1,2,3,\ldots,N_\eta\}$ and the observed log return at t_i is r_{ti}. Hence, the study focused on modelling $\{(t_i, r_{ti})\}$ which jointly form a two-dimensional Poisson process with the measure of the intensity given by

$$\Lambda[(D_2, D_1) \times (r, \infty)] = \frac{D_2-D_1}{D}\left[1-\frac{k(r-\beta)}{\alpha}\right]^{\frac{1}{k}}, \tag{12}$$

where,

$0 \leq D_1 \leq D_2 \leq T, r > \eta, \alpha > 0$

β Location parameter

α scale parameter

k shape parameter

T total number of observations

D baseline interval set to 252 trading days

η set threshold

Λ, lambda representing rate parameter for poisson distribution

The intensity measure can be represented as,

$$\Lambda[(\mathrm{D}_2, \mathrm{D}_1) \times (\mathrm{r}, \infty)] = \int_{D_1}^{D_2} \int_{r}^{\infty} \frac{1}{D} [\frac{1}{\alpha} \left(1 - \frac{k(r-\beta)}{\alpha}\right)^{\frac{1}{k}-1}] \, dtdr \qquad (13)$$

The joint distribution of function of the two-dimensional non-homogeneous Poisson process based on the extreme value for the times of the occurrence of the exceedances and the returns observed at those particular times $\{(\mathrm{t_i}, \mathrm{r_{ti}})\}$ over the two-dimensional space $[0, N] \times (\eta, \infty)$ becomes,

$$\frac{1}{D} * \frac{1}{\alpha} \left(1 - \frac{k(r-\beta)}{\alpha}\right)^{\frac{1}{k}-1} \exp\left\{-\left[1 - \frac{k(\eta-\beta)}{\alpha}\right]^{\frac{1}{k}}\right\} drdt \qquad (14)$$

where

α scale parameter

β location parameter

k shape parameter

D baseline interval

3.8.2 Maximum likelihood Estimation

The appropriate threshold for the models was determined the as in section 3.7. The data of exceedances over the threshold was apply to given the estimates of the coefficients of the explanatory variables of the parameters of the model using MLE and optimization estimation method.

The maximum likelihood function for the probability distribution in equation (14) is given by;

$$L(k, \alpha, \beta) = \left(\prod_{i=1}^{N_\eta} \frac{1}{D} \frac{1}{\alpha} \left(1 - \frac{k(\mathrm{r_{ti}}-\beta)}{\alpha}\right)^{\frac{1}{k}-1}\right) \times \exp\left[-\frac{T}{D}\left[1 - \frac{k(\eta-\beta)}{\alpha}\right]^{\frac{1}{k}}\right] \qquad (15)$$

Where

α scale parameter

β location parameter

k shape parameter

D baseline interval

T total number of observations

The measure of the intensity shows that the number of returns that exceed the threshold is proportional to the length$[D_1,\ D_2]$. The process in equation (15) assumes that the parameters $\alpha > 0, \beta,$ and k are time-invariant and therefore homogeneous. The homogeneous model is inadequate; thus, the need provide a model where the parameters could be allowed to change with time, which formed the basis for this study. The extreme value approach to value at risk considers the available explanatory variables at time t, which are incorporated into the model and used to fit a new model to the data. The parameters of this new Poisson process model change with time based on the properties of the return series and the availability of explanatory variables. This gives rise to the two-dimensional non-homogeneous Poisson process model, which was used to improve the accuracy of the parameters and provide a better estimate of the value at risk.

We assumed that the vector of explanatory variables denoted as $x_t = (x_{1t}, x_{2t}, \dots, x_{vt})$ are observed prior to time t. These variables could include factors such as volatility, interest rates and time trends. The study hence, assumed that the parameters $\alpha, \beta,$ and k are time-dependent and can be expressed as linear functions of these explanatory variables. Given that these variables are available and observed, the parameters of the model are expressed as,

$$k_t = \gamma_0 + \gamma_1 x_{1t} + \cdots + \gamma_v x_{vt} \qquad (16)$$

$$\ln(\alpha_t) = \delta_0 + \delta_1 x_{1t} + \cdots + \delta_v x_{vt} \qquad (17)$$

$$\beta_t = \theta_0 + \theta_1 x_{1t} + \cdots + \theta_v x_{vt} \qquad (18)$$

Where,

$\gamma_{i,} \delta_i$ and θ_i, for $i = 0,1, \dots, v$ are coefficients of the explanatory variables

For the analysis, two explanatory variables were chosen: volatility and interest rates. Volatility and interest rates were selected as explanatory variables in this study due to their significant influence on the behavior of extreme events in financial markets. Both variables encapsulate core aspects of market dynamics that directly affect the distributional characteristics of extreme returns. Volatility, estimated through the

GARCH (1, 1) framework, reflects the prevailing uncertainty and risk sentiment in the market, which tends to increase the frequency and magnitude of tail events during turbulent periods. Therefore, it serves as a natural predictor for the location (threshold exceedance level), scale (magnitude dispersion), and shape (tail heaviness) parameters of the EVT distribution. Interest rates, on the other hand, represent macroeconomic conditions and monetary policy stances that influence asset valuations and investor behavior. Fluctuations in interest rates affect liquidity, risk premiums, and capital flows, thereby indirectly impacting the severity and likelihood of extreme outcomes

The study also assumed that relationship between the observed interest rates, volatility and the parameters (shape, location, scale) of the model was linear.

Such that,

x_{1t} is the volatility and

x_{2t} is the interest rate

The parameters as linear function of the explanatory variables now becomes,

$$k_t = \gamma_0 + \gamma_1 \text{volatility} + \gamma_2 \text{ interest rate} \quad (19)$$

$$\ln(\alpha_t) = \delta_0 + \delta_1 \text{ volatility} + \delta_2 \text{ interest rate} \quad (20)$$

$$\beta_t = \theta_0 + \theta_1 \text{ volatility} + \theta_2 \text{ interest rate} \quad (21)$$

When the parameters are time-varying, the intensity measure of the Poisson process becomes,

$$\Lambda[(D_2, D_1) \times (r, \infty)] = \frac{D_2 - D_1}{D}\left[1 - \frac{k_t(r - \beta_t)}{\alpha_t}\right]^{\frac{1}{k_t}} \quad (22)$$

$0 \leq D_1 \leq D_2 \leq T$

The likelihood function in equation (15) becomes,

$$L(k, \alpha, \beta) = \left(\prod_{i=1}^{N_\eta} \frac{1}{D} \frac{1}{\alpha_{ti}}\left(1 - \frac{k_{ti}(r_{ti} - \beta_{ti})}{\alpha_{ti}}\right)^{\frac{1}{k_{ti}} - 1}\right) \times \exp\left[-\frac{1}{D}\sum_{t=1}^{T}\left[1 - \frac{k_t(\eta - \beta_t)}{\alpha_t}\right]^{\frac{1}{k_t}}\right] \quad (23)$$

,the log likelihood function is given by,

$$\log L(k,\alpha,\beta) = \sum_{i=1}^{N_\eta}\left[-\log D - \log\alpha_{ti} + \left(\frac{1}{k_{ti}} - 1\right)\log\left(1 - \frac{k_{ti}(\mathrm{r_{ti}} - \beta_{ti})}{\alpha_{ti}}\right)\right] - \frac{1}{D}\sum_{t=1}^{T}\left[1 - \frac{\mathrm{k_t}(\eta - \beta_\mathrm{t})}{\alpha_\mathrm{t}}\right]^{\frac{1}{\mathrm{k_t}}} \quad (24)$$

Where;

α_t scale parameter

β_t location parameter

k_t shape parameter

D baseline interval

T total number of observations

Hence, we shall use these parameters to estimate the value at risk. The value at risk is estimated using the below formula,

$$\mathrm{VaR} = \begin{cases} \beta + \frac{\alpha}{k}\{1 - [-D\ln(1-p)]^{k}\} \text{ if } k \neq 0 \\ \beta + \alpha\ln[-D\ln(1-p)] \quad \text{if } k = 0 \end{cases} \quad (25)$$

3.9 GARCH model for volatility modeling

In this study, the volatility was not directly observed, as was the case with interest rates, but it was modelled and calculated from the returns data. To model time-varying financial market volatility, this study adopted the Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model of order (1, 1), as it has proven to be effective to accurately capture volatility clustering and robust in high-frequency financial time series. The GARCH (1, 1) model accommodates the dynamic nature of conditional variance, allowing historical volatility to inform present volatility levels. This is particularly relevant when dealing with risk-sensitive frameworks, such as those involving extreme value behavior. The GARCH for a log return r_t and the model is given by,

$$r_t = \mu_t + y_t \quad \ldots\ldots(26)$$

$$y_t = \sigma_t\varepsilon_t \quad \ldots\ldots(27)$$

Where

$\varepsilon_t \sim iid\ N(0,1)$

The general GARCH (u,v) model is given by equation,

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{u} \alpha_i\, y_{t-1}^2 + \sum_{j=1}^{v} \beta_j\, \sigma_{t-1}^2 \qquad (28)$$

Where,

$\alpha_0 > 0, \alpha_i \geq 0, i = 1,2, \dots, u \quad \beta_j \geq 0, j = 1,2, \dots, v. u > 0, v \geq 0$

$$\sum_{i=1}^{u} \alpha_i + \sum_{j=1}^{v} \beta_j < 1 \qquad (29)$$

r_t is the log return value at time t

μ_t is the mean equation at time t

$\sigma_t^2$is the square of volatility at time t

ε_t is the error term with student t distribution

y_t the mean-corrected return at time t

The GARCH (1,1) model has been proven in numerous literature as the best model for volatility modeling and estimation and is given by,

$$\sigma_t^2 = \alpha_0 + \alpha_1 y_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \qquad (30)$$

Where,

$\alpha_0 > 0, \alpha_1 \geq 0, \beta_1 \leq 1, (\alpha_1 + \beta_1) < 1$

α_0 is constant term coefficient

α_1 cofficient for mean corrected return at time $t - 1$

β_1 coefficient for square of volatility at time $t - 1$

σ_t^2 Square of volatility at time t

ε_t error term with student t distribution

y_t mean- corrected return at time t

3.10 Value at Risk Estimation

In practice, for financial data, we assume $k \neq 0$ since the exceedances distribution over the threshold depicts the generalized Pareto distribution. Thus,

$$\mathrm{VaR} = \beta + \frac{\alpha}{k}\left\{1 - [-D\ln(1-p)]^{k}\right\} \qquad (31)$$

The estimates of the VaR are checked under various conditions, including different values of the probability alp.ha.

CHAPTER FOUR

RESULTS AND INTEPRETATION

4.1 Data description

The data utilized are derived from the NSE 20 share index and CBK interest rates including 2503 observations. Daily volatility for the same period was also calculated using GARCH (1, 1) model and incorporated in the analysis. Daily data was chosen to offer a detailed view of market structure and movements capturing short-term fluctuations and extreme values, which are essential for accurate Value at Risk (VaR) calculations. This large span provides sufficient observations for analysis, allowing for reliable and accurate results. This period was also selected to capture various market conditions, including both bull and bear markets, which is crucial for a comprehensive analysis of extreme market events. The NSE 20 share index is a stock market index representing 20 highest-performing among the listed companies on the NSE. The purpose of using the NSE 20 share index data is that it provides an accurate benchmark for investors and researchers to assess the general market performance, thereby serving as the primary data set for estimating value at risk which is a measure of market risk.

4.2 Statistical properties and diagonistic of returns

4.2.1 Time series plots of the data, and log returns

Figure 4.1 below shows the time series plot of the NSE 20 share index between January 2014 and December 2023.

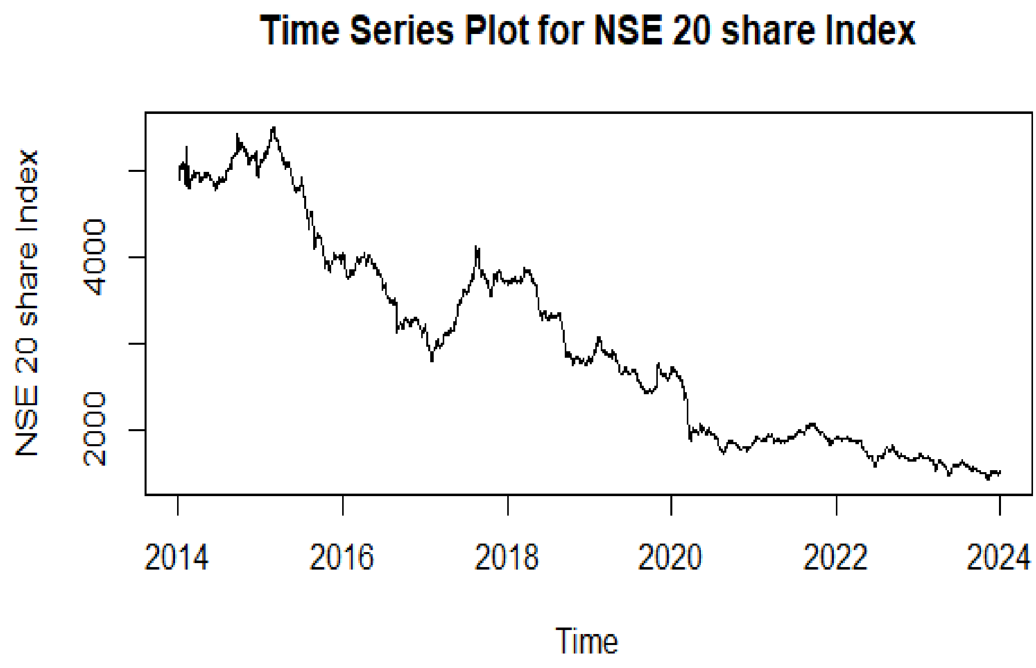

Figure 1: Time series plot for the NSE 20 share index

Figure 1 indicates that there has been an overall downward trend for the NSE 20 share index for the period, indicating a declining value in the performance of NSE during the observed period. There was a large drop in 2017 followed by a sharp increase in 2018. The NSE share index, however, continued to drop, with a significant decline in 2020. The volatility clustering is evident from the figure 1, lot as the substantial large (up-moves or down- by market's performance (up-moves or down-moves).

In our analysis, the log daily returns of the NSE 20 share index were used.

The below formula defines the log returns,

$$r_t = \ln \frac{p_t}{p_{t-1}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (31)$$

Where;

r_t daily log returns

p_t value of the NSE 20 share index at the time t

p_{t-1} value of the NSE 20 share index at the time $t-1$

The figure 2 shows the plot of the log returns for the NSE 20 share index against time.

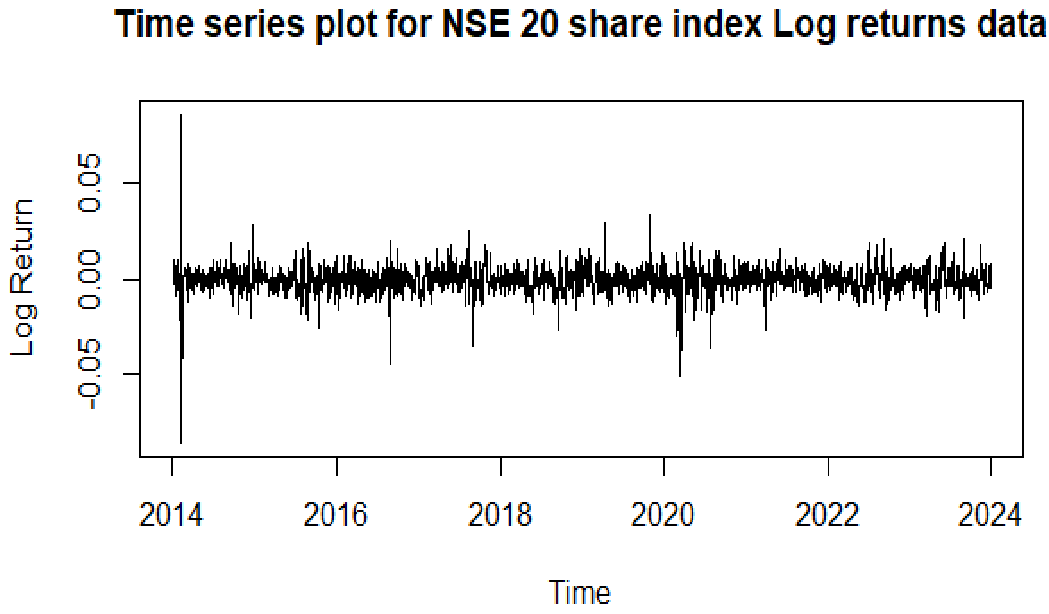

Figure 2: Time series plot for the NSE 20 share index log returns data.

The log return time series plot further illustrates the volatility clustering, where significant drops or gains follow large movements. There were sharp movements in early 2014, 2017, and 2020. This indicates the presence of extreme value in the return data. The indication of the presence of extreme events in returns data further validates the use of EVT in this research. We combine those extreme events and their occurrence times to formulate a two-dimensional Poisson process, which is used in determining the value at risk in this research.

4.2.2 Summary statistics for log returns data

Table 2 gives the descriptive statistics of the log returns of the NSE 20 share index data.

Table 2: Summary statistics of log returns series

STATISTICS	NSE 20 share index log return value
Mean	-0.000474
Median	-0.000397
Variance	0.000049
Standard Deviation	0.006989
Skewness	-0.496171
Kurtosis	21.942656
Minimum	-0.086022
Maximum	0.086344

Table 2 shows the calculated summary statistics of the data. The mean of the log returns is -0.000474 indicating overall loss during the observation period, which is close to zero hence not significant. There is a high standard deviation of 0.006989 compared to the mean, indicating high volatility in the returns, which is a property of returns data. There was a negative skewness of -0.496171, indicating that the distribution of returns is not symmetrical but exhibits asymmetry, contains heavy tails, and is skewed to the left. This is because a long-left tail in the distribution was observed compared to the right tail. This indicated that likelihood of extreme events in the data is high as compared to the case of normal distribution. The returns data has a high kurtosis of 21.942656 indicating a high peak in the distribution of the returns. This indicates that high values in the returns series are likely to be observed of either negative or positive sign.

4.2.3 Normality Test

Q-Q plot for the returns data

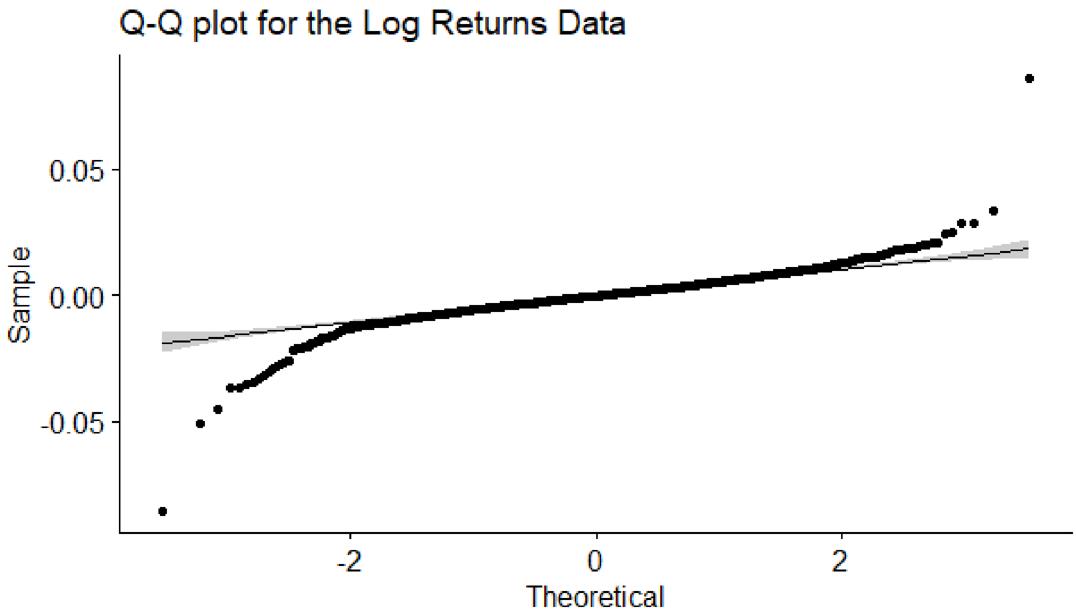

Figure 3: Q-Q plot for the returns data series

Figure 3 shows a comparison of the quantile plot for the returns data and the theoretical normal quantile plot. The Q-Q plot of the data does not depict a straight line, unlike the theoretical normal Q-Q plot. Hence, we conclude that the distribution of the returns was not normally distributed but exhibited a distribution with heavy tails and high peakedness.

Table 3: Shapiro -Wilk's test

Test statistic	p-value
W: 0.8882	**< 2.2e-16**

The null hypothesis for normality is rejected since the p-value is approximately zero. Hence, we conclude that the returns series data exhibit a distribution that is not normal.

4.2.4 Stationarity test

The **Augmented Dickey-Fuller (ADF) test** was used to test for stationarity in the returns series data

In this analysis, the data were converted to log returns to make them more stationary and stable for analysis, thereby stabilizing the variance and mean of returns over time.

The hypothesis to be tested

H_0: The log returns series is non − Stationary

H_1: The log returns series is Stationary

Table 4: Augmented Dickey-Fuller (ADF) test

Test statistic	p-value
Dickey-Fuller = -12.44	0.01

The null hypothesis of non-stationarity is rejected and concluded that the returns data series is stationary. The conversion of the data to log returns achieved the objective of making the data stationary for analysis. This consistency is crucial because many econometric models, including ARIMA, GARCH, and linear regression, rely on the assumption that the underlying data is stationary.

4.2.5 Serial correlation test

The ACF and the PACF functions were plotted to analyze the serial correlations and temporal dependencies in the returns series data.

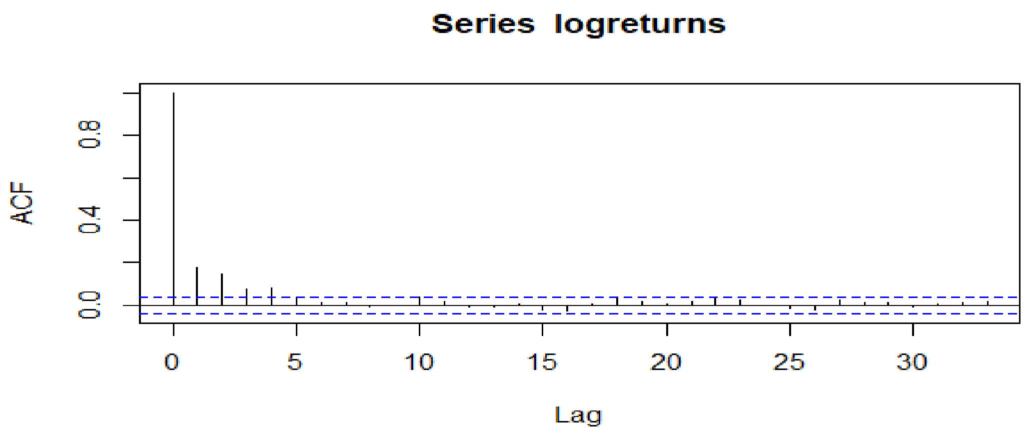

Figure 4: Autocorrelation function of the return's series

The autocorrelation function quantifies of the correlation between the returns and their lagged values. There are high and significant spikes at lags 0, 1, 2, 3, and 4, indicating high serial correlation. This indicates short-term dependencies between the data of the returns hence we conclude that serial correlation exists.

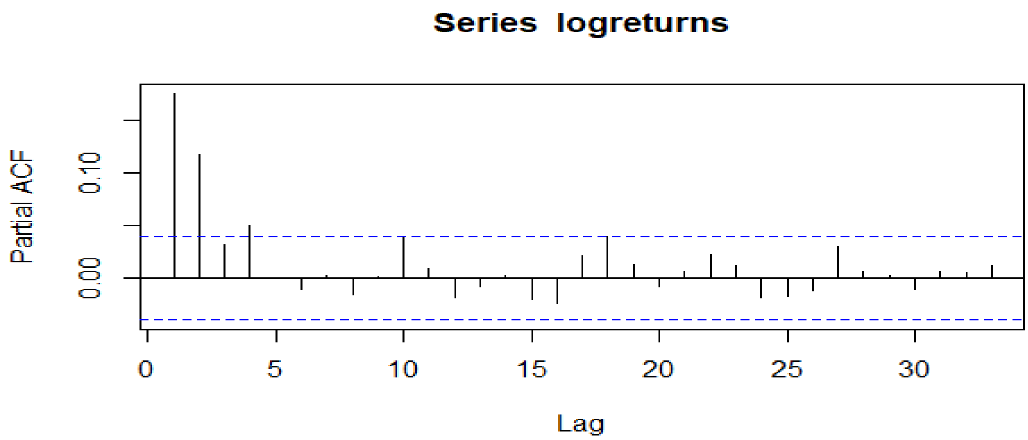

Figure 5: Partial autocorrelation function of the returns data series.

The PACF measures the correlation between the return series and its lag, adjusting for the effects of the intermediate lags. The PACF shows high series correlation for lags 1, 2, and 4 indicating short-term serial correlations.

4.2.6 Testing for ARCH Effects

In this section, the presence of ARCH effects in the returns series was tested using the ARCH LM test. These effects indicate that the variance of the residuals (errors) in the return series is not stationary, instead, it varies with time, often clustering during periods of increased volatility in the returns data.

The hypothesis to be tested

H_0: There are no ARCH effects in residuals of the return series

H_1: There are ARCH effects in residuals the return series

Table 5: Test for ARCH Effects

Test statistic	p-value
Chi-squared = 681.6	p-value < 2.2e-16

The null hypothesis is rejected and we conclude that there are ARCH effects, indicating that the variance of the residuals (errors) in the return series is not static, instead, it varies with time, often clustering during periods of increased volatility. Hence, we concluded that to account for the time-changing nature of volatility, GARCH models were appropriate for volatility estimation.

4.3 The Generalized Pareto distribution model fitting

In this section, the standard Generalized Pareto Distribution (GPD) model was fitted to the return series data, and the parameters of the GPD model were estimated. The steps for the analysis are outlined in the below sections.

4.3.1 Threshold determination

The selection of an appropriate threshold was crucial in this study since it is crucial in the accurate modelling of the tail of the distribution of the returns data series. It also ensured that, the extreme values above it are rare enough to be modelled effectively by the GPD. The quantile and the mean residual life plot methods were used in this study to determine the threshold.

4.3.1.1 Quantile method

An appropriate threshold was chosen to reduce the variance and bias of the estimates, allowing for the accurate modeling of the excess behavior using the GPD model. In this study, the quantile method was employed to determine the threshold of extreme returns for the NSE 20 share index. A threshold was selected at the 95th percentile of the stock returns data, which corresponds to the most extreme 5% of observations. Values that exceeded this threshold were considered to be extreme returns and the Generalized Pareto Distribution (GPD) was fitted to model them. The calculated threshold was 0.00944, which was appropriate and ensured that the research focused on the most significant deviations of the NSE stock returns, as it represented the tail risk, the focus of this study on VaR estimation.

Table 6: Quantile method for threshold determination

Quantile level	Threshold
95%	0.00944

4.3.1.2 Mean Residual Life (MRL) Plot method

In this approach, the threshold was established by plotting the mean excess of data points over potential thresholds. A linear MRL plot indicated that the data followed a GPD beyond that threshold.

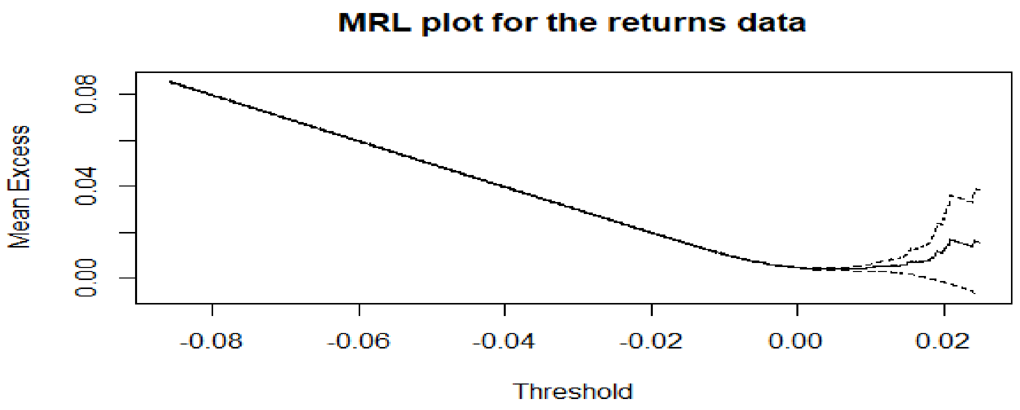

Figure 6: Mean Residual Life plot (MRL)

The Mean Residual Life (MRL) plot method was used to visually assess the suitability of different thresholds for the GPD model .The MRL plot exhibited linearity for thresholds between the 90th and 95th percentiles. Thus, the 95th percentile was selected as the threshold for the GPD model, as it balances capturing sufficient extreme data points without violating the assumption of linearity in the residuals.

4.3.2 Parameter Estimation Using Maximum Likelihood Method

The generalized Pareto distribution with the threshold level at 0.00944.The GPD was fitted to this data, and the table below provides a summary of the given model. Number of exceedances n=125

Table 7: GPD parameters

Estimated (parameter)	Value	Standard Error
Threshold (95%)	0.00944	N/A
Scale parameter	0.003342017	0.0003372725
Shape parameter	0.245104186	0.0960560579
Negative log-likelihood	-557.0063	N/A

The scale parameter measures how spread the distribution of extreme values are; in this case, the small value 0.003342017 indicates that the extreme values are not highly dispersed. The shape parameter indicated the behavior of the tail of the returns distribution. The value 0.245104186 is greater than zero, indicating a heavy-tailed distribution. This means that the returns data has a significant probability of very high extreme values.

The negative log-likelihood measured how well the data above the threshold fitted the GPD distribution. The value -557.0063, which is small, indicated a good fit for the observed excesses. The standard error for the estimated parameters were 0.0003372725 for the scale parameter and 0.0960560579 for shape parameters respectively. The low values of the standard error indicated low uncertainty and variation around the parameters, resulting in more precise estimates of the parameters.

4.4 The 2D-NHPP-based on EVT model parameter estimation using the volatility of the NSE 20 share index returns and the interest rates as the explanatory variables

The market interest rates affect the intensity of the occurrence of the extreme returns in the stock markets. Sudden changes in interest rates lead to higher uncertainty and stress in financial market, thus influencing the parameters that control the frequency of these rare events. The inclusion of interest rates as an explanatory variable in this extreme value model helps better capture the dynamics of extreme market behaviors. Volatility measures the market risk and uncertainty of market movements, which influence both the occurrence and the intensity of extreme events. For example, high-volatility periods are more likely to produce large market movements. Hence, the two explanatory variables have a significant and direct impact on the occurrence of extreme events and were therefore included in this study.

4.4.1 Time Series Plots of the Explanatory Variables

4.4.1.1 Interest Rates

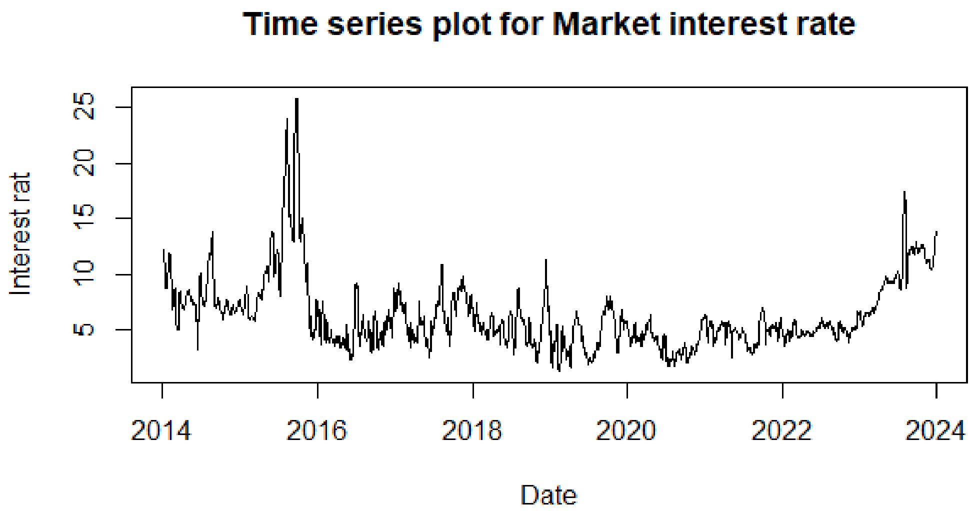

Figure 7: Time series plot for market interest rates**.**

Figure 7 shows the time series plot of market interest rates over the study period from 2014 to 2023. The plot indicates significant increases in the years between 2015 and 2016, between 2017 and 2018, in the period between 2018 and 2019 and the year 2023. The large movement influences the rate at which extreme financial events occur, as interest rates affect borrowing costs, liquidity, and overall market conditions.

4.4.1.2 Volatility of the NSE 20 share index returns

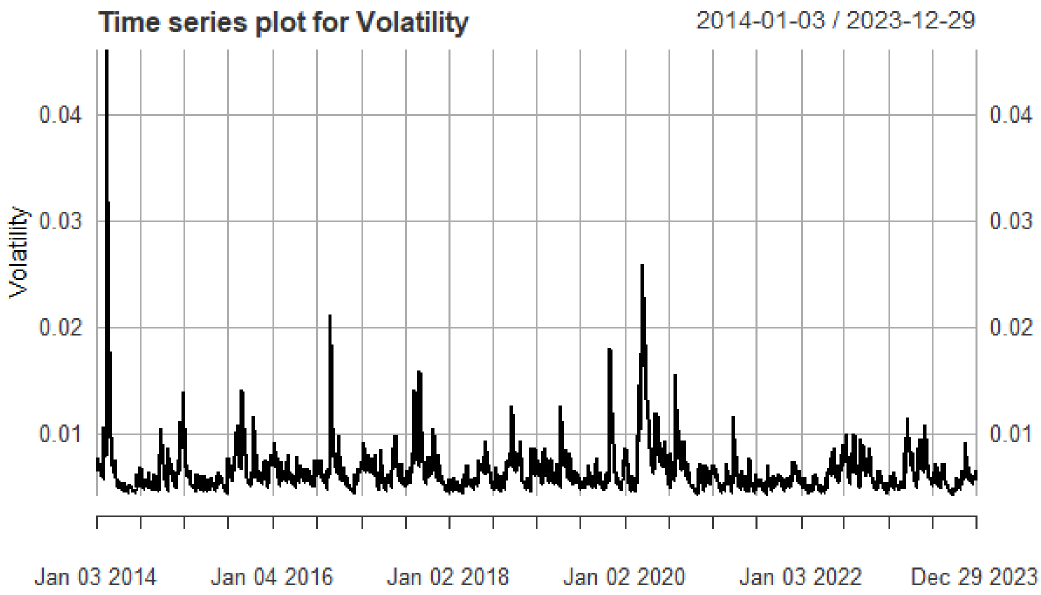

Figure 8: Time series plots for the volatility of the NSE 20 share index Returns data

Figure 8 presents the time series plot for the volatility of NSE 20 share index returns data, estimated using the GARCH (1, 1) model, over the period from 2014 to 2023. The periods between 2014 and 2015, 2016 and 2018, and 2020 and 2021 show a significant increase in volatility, with volatility clustering observed around the same periods. These periods are indicative of unstable market conditions where extreme events are more likely, thus impacting the parameters of the Poisson process. The model accounts for this time-varying volatility to predict the occurrence of rare events more accurately.

4.4.1.3 Volatility modeling using GARCH (1, 1)

The GARCH (1, 1) model was employed to capture the conditional Heteroscedasticity present in the log returns of the NSE 20 Share Index. The estimation results show that the model effectively accounts for time-varying volatility, a common feature in financial time series where large market movements tend to cluster together. In the mean equation, the constant term, μ, is estimated at -0.0002818 and is statistically significant at the 1% level ($p < .001$), with a t-value of -63.87. This negative estimate, although small in magnitude, suggests a slight downward trend in average daily returns during the study period. The narrow 95% confidence interval [−0.0002904, −0.0002731] confirms the precision of this estimate.

The volatility equation, comprising the parameters ω,α_1 and β_1, provides insight into the structure of market volatility. The estimate for ω, which represents the long-term average variance, is 0.000004846 ($\text{P value} = 0$) and is statistically significant with an extremely small standard error. This result reflects a consistent low level of variance in returns, reinforcing the suitability of the GARCH framework for modeling baseline volatility. The α_1 parameter, estimated at 0.2000 ($\text{P value} = 0$) , captures the short-term reaction of volatility to market shocks, indicating that recent squared returns (news or innovations) have a notable impact on current volatility levels. The β_1 parameter, which measures the persistence of volatility, is estimated at 0.7000 ($\text{P value} = 0$) and is highly significant. This value indicates a strong memory in the volatility process, meaning that during periods of increased volatility, it typically stays high for some period of time which is a key characteristic known as volatility clustering. Together, the sum of α_1 and β_1 equals 0.90, suggesting high but mean-reverting persistence in volatility. This means the effects of a volatility shock dissipate slowly over time, but the process remains stationary. Such findings confirm that volatility in the NSE 20 Share Index is both responsive to new information and persistent across time, making the GARCH(1,1) model an appropriate tool for volatility modeling in this context. These volatility estimates are crucial for informing the subsequent stages of this study, particularly the modeling of risk through the 2D-NHPP-based EVT framework, where conditional volatility is used as a covariate to explain the occurrence and magnitude of extreme market events.

4.4.2 Parameter Estimation of 2D-NHPP based on EVT Model

The maximum likelihood method was employed to estimate the scale, shape and location parameters given by the below forms.

$$k_t = \gamma_0 + \gamma_1 \text{volatility} + \gamma_2 \text{ interest rate}$$

$$\ln(\alpha_t) = \delta_0 + \delta_1 \text{ volatility} + \delta_2 \text{ interest rate}$$

$$\beta_t = \theta_0 + \theta_1 \text{ volatility} + \theta_2 \text{ interest rate}$$

Where, the parameters,k_t, α_t and β_t are linear functions of the volatility of the NSE 20 share index and the interest rate at time t.

Table 8 presents the results of the estimated coefficients of the parameters.

Table 8: parameter coefficients for the 2D-NHPP model

Parameter	Estimate	SE	P value
$\boldsymbol{\gamma_0}$	0.0003448139	0.00000001	0.0000
$\boldsymbol{\gamma_1}$	0.0009956535	0.00000001	0.0000
$\boldsymbol{\gamma_2}$	-0.0044906778	0.00000001	0.0000
$\boldsymbol{\delta_0}$	-5.5679734737	0.000001751139	0.0000
$\boldsymbol{\delta_1}$	62.6763138145	0.00000001054213	0.0000
$\boldsymbol{\delta_2}$	-0.0614310886	0.000008137154	0.0000
$\boldsymbol{\theta_0}$	0.0366819993	0.00007455924	0.0000
$\boldsymbol{\theta_1}$	1.2794835187	0.0000004407886	0.0000
$\boldsymbol{\theta_2}$	-0.0021601651	0.00000001434671	0.0000

The table above presents the coefficients that describe the relationship between the parameters k_t, $\ln(\alpha_t)$, β_t, and the explanatory variables: the volatility of stock returns and the interest rates. The estimated coefficients demonstrate that both volatility and interest rates play pivotal roles in shaping the risk dynamics under the two-dimensional non-homogeneous Poisson process, based on EVT. The P-values associated with all the coefficients are zero, indicating that both volatility and interest rates are significant in the linear models of the parameters (, scale, and shape, location)

Shape parameter ($\mathbf{k_t}$)

The shape parameter k_t is modelled by the equation

$k_t = 0.0003448139 + 0.0009956535 \text{ volatility} - 0.0044906778 \text{ interest rate.}$

For volatility $\boldsymbol{\gamma_1}$ $=0.0003448139$ (P value $= 0$), this indicated a positive and highly significant relationship between volatility k_t. This showed that, as the volatility increased, the shape parameter rises suggesting a heavier tail in the distribution of extreme events. This indicated that higher volatility increases the likelihood of extreme events in the stock returns data.

For the interest rates, $\boldsymbol{\gamma_2} =- 0.0044906778$ $(\text{P value} = 0)$, is negative and statistically significant. This indicates that as interest rates rise, the shape parameter decreases, suggesting a thinner tail in the distribution and reduced extremes. This finding aligns with economic theory, which suggests that higher interest rates often dampen speculative activity and risk-taking behavior, thereby reducing the likelihood of extreme financial losses.

Scale parameter ln $(\boldsymbol{\alpha_t})$

The log- scale parameter is given by,

$\ln(\alpha_t) =- 5.5679734737+$
$62.6763138145\ \text{volatility} - 0.0614310886\ \text{interest rate}$

The coefficient for volatility, $\boldsymbol{\delta_1} = 62.6763138145$ $(\text{P value} = 0)$ is positive and statistically significant, indicating that increased volatility leads to a larger scale parameter. This suggests that during periods of high market volatility, the magnitude of extreme events grows, which is consistent with the behavior of financial markets during turbulent times.

The coefficient for interest rates, $\boldsymbol{\delta_2} =- 0.0614310886$ $(\text{P value} = 0)$ is negative and significant. This implies that higher interest rates mitigate the scale of extreme losses, reflecting a stabilizing effect on the financial system. The statistical significance of both coefficients underscores the robustness of these relationships, providing strong evidence for their role in influencing the scale of risk.

Location parameter $\boldsymbol{\beta_t}$

The location parameter is given by the equation,

$\boldsymbol{\beta_t} = 0.0366819993 + 1.2794835187\ \text{volatility} - 0.0021601651\ \text{interest rate}$

The coefficient for volatility, $\theta_1 = 1.2794835187$ $(\text{P value} = 0)$ suggesting that higher volatility significantly increases the baseline level of risk. This finding aligns with the intuition that market instability amplifies baseline losses.

The coefficient for volatility, $\theta_2 =- 0.0021601651$ $(\text{P value} = 0)$ is negative and statistically significant. This indicates that as interest rates rise, the baseline level of risk decreases. These results suggest that monetary policy interventions, such as higher interest rates, can effectively mitigate financial instability.

4.4.3 Performance of parameters over time

The performance of the parameters over time with relation to the observed explanatory variables is showed in the below graph.

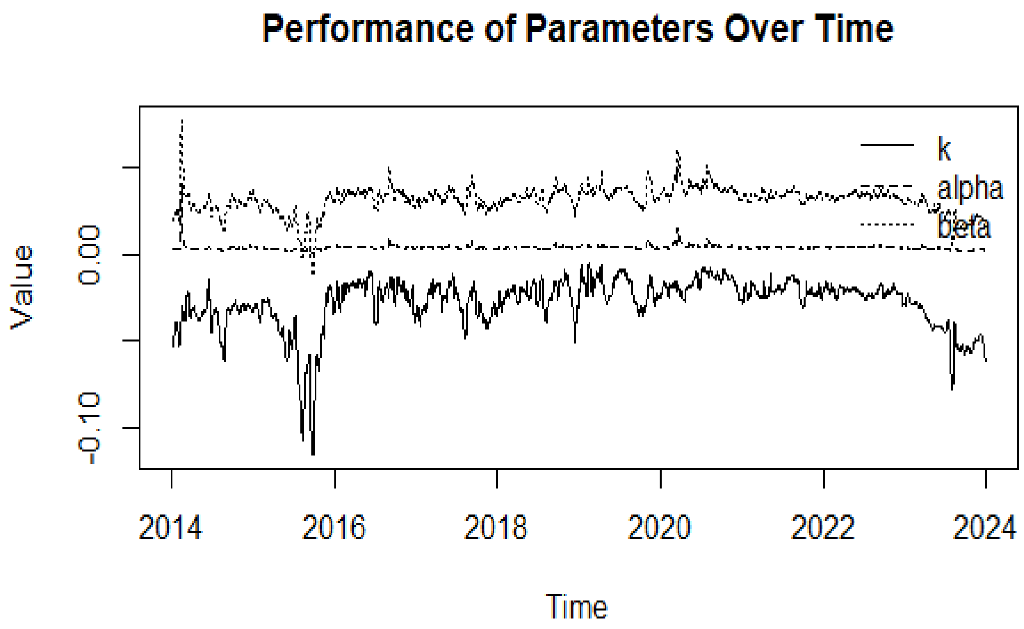

Figure 9: Time series plot for the performance of parameters over the observed period

The location parameter β_t remained relatively stable over the observed period, indicating consistency in the model's central tendency. This stability suggests that β_t is less sensitive to short-term market fluctuations or external shocks.

The scale parameter α_t, which represents variability or dispersion, showed significant fluctuations over time. These fluctuations reflected periods of heightened market volatility or changes in external economic conditions. For instance, sharp spikes in α_t correspond to periods of increased uncertainty in the dataset.

The shape parameter k_t exhibited pronounced sensitivity to changes over time, with frequent peaks and troughs. This variability indicated that the model's tail behavior is highly reactive to shifts in the underlying financial conditions, such as extreme returns or outlier events.

4.5 Value at risk (VaR) Estimation

The data were analyzed using daily observations; hence, the daily value at risk was estimated. According to the observed values on 29 December 2023, the volatility was 0.00647021, and the interest rate was 13.7786.

Therefore, the parameters are were estimated as, $k_t = -0.05842, \alpha_t = 0.00245684$ and $\beta_t = 0.0151965$

The estimated shape parameter (k_t) for the non-homogeneous Poisson process is negative, indicating a bounded tail distribution. This suggested that extreme losses are capped under the modeled conditions. The negative relationship between k_t and interest rates, as evidenced by the statistically significant coefficient $\boldsymbol{\gamma_2} = -0.0044906778$ ($\text{P value} = 0$), reflects the stabilizing role of higher interest rates. Elevated interest rates typically deter speculative investments and leverage, which mitigates the occurrence of extreme financial events. This finding is consistent with theoretical expectations and highlights the effectiveness of monetary policy in moderating systemic risk during periods of high market volatility.

$$\mathrm{VaR} = \beta + \frac{\alpha}{k}\{1 - [-D\ln(1-p)]^k\}$$

The value at risk at various confidence levels is given by,

Table 9: Value at risk at different levels of confidence

Confidence level	Value at Risk (VaR)
90%	0.01319653
95%	0.01258546
99%	0.01160697

The VaR indicates the potential losses at the 90%, 95%, and 99% levels of confidence as 0.01319653, 0.01258546, and 0.01160697, respectively, for every unit of investment in this case, which is kshs. The VaR values decreased as the confidence level increased, which align with Value at Risk (VaR) definition. At higher confidence levels, the model estimates smaller potential losses because extreme events are assumed to be less probable. The results also show that the bounded tail distribution effectively limits extreme losses, aligning with the negative shape parameter $k_t = -0.05842$.

CHAPTER FIVE

DISCUSSION, CONCLUSION AND RECOMMENDATION

5.1 Discussion

The study employed a two-dimensional non-homogeneous Poisson process based on an EVT model to provide a more accurate measure of financial risk, focusing specifically on the value at risk. This study aimed to estimate Value at Risk (VaR) using a two-dimensional non-homogeneous Poisson process (NHPP) model, incorporating volatility and interest rates as explanatory variables for the shape, scale, and location parameters of the model. The motivation behind this approach was to understand the contribution of the volatility and interest rates variables and the risk they impart on the system. In particular, we sought to determine how volatility and interest rates affect the behavior of the tails of the distribution and how these relationships impact the estimation of Value at Risk (VaR).

The study started with analyzing the statistical characteristics of the NSE 20 share index return series. The data were transformed into returns and then into log returns for analysis. The summary statistics indicated that the return series had a negative mean, indicating an overall loss in the chosen period. The data had a negative skewness and also high kurtosis, indicating a departure from the normal distribution. The test for normality indicated that the return series exhibited a heavy tailed and high pealed distribution, but showed a distribution with heavy tails, hence an increased likelihood of extreme returns. The data exhibited serial correlation between the data returns of lags 0, 1, 2, 3, and 4, indicating short-term dependencies between the data returns. There was presence of ARCH effects, indicating that the variance of the residuals (errors) in the return series is not static, instead, it varies with time, often clustering during periods of high volatility.

A threshold was selected at 95% level (0.00944) using the quantile method and the mean residual life plot method (MRL). It provided sufficient extreme events for fitting the GPD, offering a balance between variance and bias. Results of this study highlighted a clear and intuitive relationship between the explanatory variables and the model parameters. The volatility variable was found to be positively related to all three model parameters shape, scale, and location. This result aligns with economic theory, which suggests that increased volatility in financial markets corresponds to a higher likelihood of extreme events (i.e., larger deviations from the mean). Volatility represents the uncertainty or risk in the market, and naturally, higher volatility increases the probability of extreme outcomes, thus affecting the distribution's tail and, consequently, the VaR estimation. Conversely, the relationship between interest rates and the model parameters was negative. This suggests that as interest rates rise, the likelihood of extreme negative events, such as, large financial losses decreases. This negative relationship aligns with traditional economic theory which postulate that higher interest rates typically reduce the demand for credit and risk-taking behaviors in the market, which, in turn, reduces the potential for large, extreme events. This inverse relationship suggests that financial markets tend to behave more conservatively as interest rates rise, thereby reducing the risk of extreme events.

In calculating VaR, observed interest rates and calculated volatility were used to estimate the potential risk to the system. The VaR values decreased as the confidence level increased, which align with Value at Risk (VaR) definition. At higher confidence levels, the model estimates smaller potential losses because extreme events are assumed to be less probable. By applying the two-dimensional NHPP model, the study was able to provide more dynamic and time-varying estimates of risk, in contrast to traditional methods that assume a static risk profile. This approach is particularly useful in volatile markets where the risk profile fluctuates over time, and where traditional methods might fail to capture these fluctuations accurately.

5.2 Conclusions

This study demonstrated the effectiveness of using a two-dimensional non-homogeneous Poisson process (NHPP) for estimating Value at Risk (VaR) by incorporating volatility and interest rates as explanatory variables for the shape, scale, and location parameters of the model. The findings revealed important insights into the relationships between these financial variables and the risk in the system. This provided improved and robust methods of providing a more accurate and reliable measurement of value at risk. By allowing the model parameters with time, the study was able to update the parameters with new observed explanatory variables hence more updated and accurate measure of financial risk.

The volatility's positive relationship with VaR parameters which indicated that as volatility increases, so do the values of the model parameters (shape, scale, and location), supporting the theory that increased volatility leads to greater potential for extreme events in the financial market, thereby increasing the risk as measured by VaR. The interest rates' negative relationship with VaR parameters which indicated that as interest rates rise, the likelihood of extreme negative outcomes decreases, highlighting the stabilizing effect of higher interest rates on the financial markets. The interest rates decisions are interventions made by financial systems and central banks to protect the market against adverse losses in periods of market instability and high volatility, which is in line with the conclusions of this study. The study provided dynamic risk estimation with NHPP as compare to static risk estimation provide by traditional methods by the use of a non-homogeneous Poisson process model, which accommodates time-varying risk factors, provides a more realistic and flexible approach to estimating VaR, particularly in volatile market conditions. The study also provides a more practical implications for risk since this approach enables more accurate risk assessment, helping financial institutions, policymakers and investors to make better- informed decisions regarding risk exposure, asset allocation, and market behavior.

5.3 Recommendations

The below recommendations have been made for future research and practical risk mitigation and management applications.

Further exploration of the relationship between calculated volatility of returns and observed interest rates and, that is, future studies should explore the correlation between the volatility of returns and observed interest rates as this relationship can significantly impact risk estimation. Accounting for this correlation in the NHPP model may improve the accuracy of VaR estimates and offer a broader view of the risks in the financial system.

The study also recommends improvement of threshold selection in Extreme Value Analysis. The accuracy of the EVT models, particularly the Generalized Pareto Distribution (GPD), relies heavily on the threshold chosen in the Peaks over Threshold

(POT) method. Future research should focus on developing more robust methods for selecting the threshold.

Incorporating Additional Macroeconomic Factors: While volatility and interest rates were used as the primary explanatory variables, other macroeconomic factors (such as inflation rates, unemployment levels, or global events) could also affect financial risk. Future studies could expand the model to include these additional variables, which would allow for a more comprehensive risk assessment.

Expanding the Model to Multi-Asset Portfolios: This study focused on a single asset or portfolio, but financial markets typically involve multi-asset portfolios with multiple risk factors. Extending the NHPP model to include multiple assets would improve the accuracy of VaR estimates for diversified portfolios and offer more reliable insights into systemic risk.

Further Validation of the Model across Different Market Conditions: It is essential to test and validate the model across different market regimes, especially during periods of financial crisis or extreme events. This would help to assess the robustness of the model and its ability to accurately estimate VaR in times of market stress.

5.4 Limitations of the study

However, while the use of the NHPP model provides a dynamic framework for estimating risk, it is not without limitations. One such limitation is the assumption that volatility and interest rates are independent variables. In reality, these two variables are often correlated, and ignoring this correlation could lead to less accurate estimates of the model parameters.

The study relies on a limited set of explanatory variables, specifically volatility and interest rates. Although these are significant macro-financial indicators, excluding other relevant variables such as liquidity measures, inflation expectations, market sentiment indices, or geopolitical risks may lead to omitted variable bias. These excluded factors might contribute meaningfully to extreme market movements, thus affecting the accuracy of estimated VaR levels.

The model's performance and estimation are data-sensitive, particularly in relation to the threshold selection for extracting extreme values. EVT models, including the GPD framework, rely on accurate threshold choices to define exceedances. An inappropriate threshold can either include too many observations (reducing the extremeness of the sample) or too few (leading to estimation instability). The study assumes a fixed threshold, which may not optimally adapt to evolving market conditions.

While the two-dimensional non-homogeneous Poisson process captures temporal and conditional variations in extreme events, it assumes independence of occurrences within time intervals and homogeneity across dimensions apart from the covariates. This may not fully reflect clustering effects or contagion often observed during financial crises, where extreme events tend to occur in bursts and exhibit temporal dependence

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