MATHEMATICAL MODELING OF DRUG ABUSE, UNEMPLOYMENT AND MENTAL STRESS ON POPULATION DYNAMICS OF MENTAL ILLNESS

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DECLARATION

This project is my original work and has not been presented elsewhere for a degree or any other award.

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DEDICATION

I dedicate my project to my beloved wife Doreen Wanjiru, my mother Catherine Ngina, my siblings Mercy Mueni, Nichodemus Ngila, Cecilia Nthenya, Ann Nduku and finally my great and very supportive friend Stanley Mutemi Muithya for their unwavering support and encouragement during the journey. You have been my constant cheerleaders through every academic and personal endeavor in my life.

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ABBREVIATION AND ACRONYMS

COVID-19	Corona Virus Disease 2019
D.A	Drug Abuse
M.S	Mental Stress
NCD	Non-Communicable Diseases
PET	Positron Emission Tomography
ODE	Ordinary Differential Equations
SIR	Susceptible-Infectious-Recovered
SEIR	Susceptible-Exposed-Infected-Recovered
SIS	Susceptible-Infected-Susceptible
VCIND	Vascular Cognitive Impairment
VTE	Venous Thromboembolism
WHO	World Health Organization

DEFINITION OF OPERATIONAL TERMS

Compartmental Models:	Model that classifies individuals into different sub populations or compartments based on certain characteristics connected by the migration or transformation of individuals and governed by nonlinear ordinary differential equations system that tracks the population as a function of time.
Heterogeneous Population:	A population where individuals have different value for characteristic curve (they are not similar to one another). For example, you could have a heterogeneous population in terms of humans that have migrated from different regions of the world and currently live together.
Simulation:	Performing an experiment using a computer.
Mathematical modelling:	Mathematical language of explaining a real world phenomenon by considering a set of assumptions.
Stability:	Consistency and ability of a system to converge when perturbed.
Stability analysis:	An acceptable tool for studying long-term behavior of dynamical systems

LIST OF GREEK SYMBOLS

α	alpha	Transmission rate from employed population with M.S.
β	beta	Transmission rate from unemployed population with M.S.
ε	epsilon	Rate of natural death.
λ	lambda	Rate of recruitment.
μ	mu	Rate of recovery.
ω	omega	Transmission rate from susceptible class.
π	pi	Transmission rate from unemployed population on D.A.
ψ	psi	Group of parameters.
ρ	rho	Transmission rate from employed population on D.A.
τ	tau	Transmission rate for the unemployed.
θ	theta	Transmission rate for the employed.

ABSTRACT

There has been a rise in the number of reported cases of mental illness in both High Income Countries (HICs) and Low and Middle Income Countries (LMICs). Non-communicable Diseases (NCDs) seldom make use of mathematical modeling. This research suggests eight first-order differential equations to form the basis of a mathematical model for psychiatric disorders. There are eight distinct categories created to reflect the public at large: the vulnerable, the working and jobless, drug addicts, the emotionally distraught, and the mentally ill. Theoretically, the well-posedness of the model equations is established by examining the positive, bounded, existing, unique solutions and the local and global stability. The eigenvalue approach was used to investigate local stability, and a Lyapunov function was created to analyze global behavior. In order to back up the analytical results, we performed a numerical investigation of the dynamical behavior of the model's equations using the fourth-order Runge-Kutta technique with the use of the MATLAB software package. To better understand the impact of environmental factors on mental disease, researchers have experimented with changing a number of variables related to mental stress, unemployment and drug addiction among certain groups. Based on the findings, the prevalence of mental illness skyrocketed anytime variables related to psychological strain or substance (drug) addiction rose in severity. In conclusion, lowering the growing rates of mental illness may be accomplished through increasing options for employment, improving working conditions, and fostering a welcoming workplace.

CHAPTER ONE

INTRODUCTION

1.1 Background Information

Patients, professionals, and members of the general public all have different actions and emotional responses based on how they think the world views their experiences with mental diseases. People's behavioral and emotional reactions are heavily influenced by their beliefs about the events that occur (Lobban et al., 2003). Alterations in one's emotional state, level of thought, or pattern of conduct are all symptoms of a mental disorder (or a combination of the three). They are non-communicable diseases (NCDs) related to distress and social, work, or family issues (Daud & Qing, 2021). Non-communicable diseases (NCDs) are presently the main health and development threat to people worldwide. Currently, the stratified heterogeneity of NCD fatalities is seldom addressed (Wang & Wang, 2020).

Through several studies conducted in High-Income Countries (HIC), there has been an adoption of effective systems and approaches to mental health. Several Low and Middle-Income Countries (LMIC) have attempted to address mental health challenges. The lack of enough resources and logistics to achieve a mental illness-free community remains a big challenge (Mutiso et al., 2020). Mental disorders are distressing and disturbing and pose an enormous burden in terms of cost, morbidity, and mortality, according to (Thakur & Roy, 2021), which delay the accomplishment of sustainable developmental goals for a country. Since addressing mental health is crucial to achieving universal health coverage, as outlined by the "Big Four Agenda" and necessary to realize Vision 2030, it has been thrust to the forefront of Kenyan policy priorities.

The World Health Organization (WHO) Global Mental Health Action Plan 2013-2020 alludes that mental health remains a key determinant of any country's overall health and socio-economic development. Regarding WHO, a variety of outcomes for individuals within a given society, such as healthier lifestyles; better physical health; improved recovery from illness; fewer limitations in daily living; higher education attainment; greater productivity, employment, and earnings; better family relationships; social cohesion and engagement and improved quality of life, largely depend on the state of mental health. Mathematical models have been utilized for years to study biological sciences to understand diverse aspects of non-communicable diseases such as diabetes mellitus (López-Palau & Olais-Govea, 2020).

Mathematical Modelling and simulation are effective tools for creating universal healthcare strategies (Agarwal & Pathak, 2014). In recent studies, models formulated for dynamics have informed strategic planning, implementation and evaluation of control programs (Ronoh et al., 2020). Unlike infectious illnesses, mathematical modelling is seldom utilized for NCDs, and a model that currently treats all aspects of the disease does not exist (Rosjat et al., 2014). Recent research has demonstrated that linear systems may explain and better predict NCD population dynamics (Boutayeb et al., 2004). Examining population models qualitatively has been studied (Paul et al., 2022). The population dynamics of mental diseases in this study was characterized by a set of constant coefficient ordinary differential equations (ODEs).

1.2 Statement of the problem

The rise of mental illness which is a non-communicable disease, in Kenya and globe at large poses a threat to human health and a drain to the economy. There is a prevalence quotation of 13.7% for lifetime and 10.6% for the current morbidity in mental disorders (Kwobah et al., 2017). Recently, there has been a surge in mortality rate as a result of mental illness according to World Health Organization (WHO). The prediction criterion concerning the rise of mental illness and its impact is an important public health concern in Kenya and the world in general.

In this case, Kenya as a country had experienced the adverse effects of mental illness which was more prevalent on the youth and young adults. Inadequate knowledge and information regarding mental illness stressors such as drug abuse, mental stress and unemployment has been a great limitation to the policies governing mental health awareness, treatment, and control by the health sector. This study did formulate a mathematical model on mental illness in order to provide knowledge that could be incorporated in policy frameworks geared towards mental health awareness, treatment, and control measures of mental health disorders.

1.3 Justification of the study

Mental illness is major global health burden and has adverse effects on economy. Mathematical prediction models on risk factors associated to mental diseases and their impact on the economy in Kenya and globally are rarely explored. Mental disorders vary and therefore research and identity of mental impairments affecting most of the county residents can be used to make inferences and control of mental disorders by the local county government(s) and the national government(s) at large within the global set up. No doubt, mental illness is currently in a state of flux. Rapidly increasing cases of mental illness in the counties, most of the developing countries such as Kenya and the globe at large are of great concern. Very little research on modelling has been done concerning mental illness. It is from this background; this study sought to develop a mathematical model that will predict, by examining to what extend mental stress, unemployment and drug (substance) abuse effect mental illness in a population which would be helpful in formulating policies geared towards sustainable development goals (SDG) in Kenya.

1.4 Significance of the study

The study was to provide knowledge that could be incorporated in policy frameworks geared towards mental health awareness, treatment, and control measures of mental health disorders. The study would help formulate a mathematical model on prevalence of mental illness which is among the least researched of the NCD despite the immerse effect it has on the economy. The county government's health sector is under-resourced by the national government which is not keen on enough funding on mental health care system characterized by inaccessible services, an acute shortage of mental health workers and limited funding (Jaguga & Kwobah, 2020). Modeling mental health disorders would give a great insight on attainment of universal health since good health is essential to sustainable development.

1.5 Research questions

1. What are the dynamics associated with mental illness in a study population?
2. What measures make a compartmental population based model well posed?
3. To what extend can ODE's be used to study the impact of drug abuse, unemployment and mental stress on mental illness?

1.6 Objectives of the study

This section presents the general and specific objectives of the study.

1.6.1 General objective

To model drug abuse, unemployment and mental stress on population dynamics of mental illness.

1.6.2 Specific Objectives

The specific objectives were to;

1. Formulate a compartmental population based model to study population dynamics of mental illness.
2. Analyze the population dynamic model using theories of ordinary differential equation.
3. Validate the model using secondary data.

1.7 Scope of the study

There are many risk factors associated with mental illness. This study focuses on drug (substance) abuse, unemployment and mental stress as the main contributors of mental illness. It aims at modelling the three risk factors on population dynamics of mental illness.

CHAPTER TWO

LITERATURE REVIEW

2.1 Review of Non communicable diseases

Recent studies have shown that Non communicable diseases (NCDs) are the leading causes of both mortality and morbidity worldwide (Mat Daud, 2020). The major challenges facing public heath by increasing the burden of infective diseases, especially for the developing as well as the low and middle income countries like Kenya is associated with NCDs (Mbwayo et al., 2019; Wang & Wang, 2020). Very little work has been done concerning mathematical modelling of non-communicable diseases (NCDs) compared to the infectious diseases. There are studies which have indicated linear systems to better describe and model population dynamics of NCDs (Boutayeb et al., 2004).

A general linear mathematical model of non-communicable disease (NCD) has been formulated (Daud & Qing, 2021). The research used compartmental analysis and its qualitative properties analyzed without finding the eigenvalues. The research provided proof of the qualitative properties of the general model including the existence and uniqueness of its solution and equilibrium, and the positivity and boundedness of its solutions. The global stability of the general model was analyzed using the theorem of compartmental matrix and Lyapunov function. The outcome of the model was existence of one unique non-negative equilibrium which was globally exponentially stable. As a real-world example, the general model and its qualitative analysis are implemented to a NCD, namely venous thromboembolism (VTE) among pregnant and postpartum women. VTE was selected for the study as it was a major global health burden due to its association with disability and lower quality of life and death (Daud & Qing, 2021).

To study NCDs, a number of issues such as the lack of NCDs models formulated using differential equations, wrong formulated mathematical terms as well as ill-posedness of the governing equations have previously been highlighted (Mat Daud, 2020).

There are, among many others, four NCDs studied using a mathematical model(s) in the recent past: VTE and thyroid disorder (Daud & Qing, 2021), hypertension, and diabetes mellitus for pregnant patients, which were all proposed and studied by applying the stability and sensitivity analysis (Mat Daud et al., 2019). This study attempted to develop a model and formulate well-posed ODEs (Mat Daud, 2020).

The prediction of epidemiological characteristics of vascular cognitive impairment based on susceptible infectious recovered and immune hosts (SIR) mathematical model and the effect of brain rehabilitation health measurement system on the cognitive function of patients have been developed (Hu et al., 2021). The SIR mathematical model was used to predict the epidemiological characteristics of vascular cognitive impairment (VCIND). This study aimed to investigate the effects of brain rehabilitation health care measurement system-assisted cognitive training on cognitive function and event-related potential (ERP) P300 in patients with VCIND. The study found that brain rehabilitation health care measurement system-assisted cognitive training can effectively improve patients' cognitive function and daily activities with VCIND (Hu et al., 2021).

2.2 Review of Mental Illness

It is essential to understand the impact of Serious Mental Illness (SMI) on parents and children. Parenting alongside SMI can be challenging, and supporting parents is a priority. To develop effective interventions, it is crucial to understand the experiences and support needs of these parents. A systematic review has been conducted to synthesize qualitative research on impact of Serious Mental Illness (SMI). The review identified six themes: parenting difficulties, strained parent-child relationships, and the need for wrap-around support. The findings highlight the importance of systemic practice changes and compassionate support to help parents manage their caregiving roles and improve outcomes for themselves and their children (Harries et al., 2023).

According to (Jaguga & Kwobah, 2020), the COVID-19 pandemic did exert a considerable impact on public mental health globally. With the rise of the pandemic in sub-Saharan Africa, including Kenya, there was a need to clearly describe and provide evidence to guide the mental health response in the region (Jaguga & Kwobah, 2020). This would offer context-based and specific recommendations to help improve the mental well-being response in Kenya. By using the information obtained from the official WHO website released between the 13th day of March 2020 and the 31st day of July 2020, there was no formal mental health response framework in Kenya, leading to an unfulfilled necessity for psychological first aid within the community (Jaguga & Kwobah, 2020).

Most countries used lockdowns as a containment measure during the COVID-19 pandemic, which successfully contributed to reducing the contagion. However, the related mobility restrictions lead to a considerable rise in risks of major depressive and anxiety disorders. Mental disorders burden lead to a considerable part of COVID-19-related deaths. Recent studies show that a cost-benefit analysis of movement restriction should consider a forecast of mental disorder development in the population. Despite the preparation of clear and well-thought-out guidelines for the control of mental health conditions during the pandemic, the poorly resourced psychiatric departments lead to poor implementation of policies. The lack of a mental health surveillance system negatively impacted the aptitude to design evidence-based interventions (Alexeevich et al., 2023).

In order to mitigate the deleterious effect of COVID-19 on public mental health in Kenya, four crucial strategies that can strengthen psychiatric responses were discussed: preparing mental health response plan, allocation of funds for psychiatric departments, equipping the community health workers with new skills relevant to current issues, encouraging community health volunteers on psychological well-being which will enable first aid measures at societal level as well as continuous text message surveys on the mental health impact of the COVID-19 pandemic to inform decision-making (Mbwayo et al., 2019; Wang & Wang, 2020).

Mental health plays a vital role regarding sustainable developmental goals globally, as the WHO indicated recently. A common challenge faced by the healthcare systems in developing countries is the substantial unmet mental healthcare needs or the large gap between the need for and the provision of mental healthcare treatment. There are four significant barriers to attaining appropriate health care for people with mental disorders in both developed and developing countries, namely the high nonmonetary cost as a result of stigma within the society, the high lack of funds for payment due to insufficient public funds devoted to mental health by the government(s), the high time costs due to inadequate mental healthcare resource availability as well as the low treatment benefit as a result of lack of adequate development of prediction mathematical model(s) to guide on prevention and treatment (Qin & Hsieh, 2020).

In the past decade, mental dysfunction, especially depression, has predominantly increased significantly and is listed as a significant threat to disability, with suicide listed as the second leading cause of death (Saxena et al., 2013). The majority of people with severe mental health issues die prematurely, as much as two decades early, due to preventable physical conditions they are exposed to. With the continued increase of the aging process, the incidence rate of mental disorders is increasing. The incidence rate of mental impairment with varying degrees of violence also increases rapidly. The impact of mental illness has caused great harm to patients, significantly reduced the quality of life of patients, and caused a considerable burden on many families with victims and society as well as the government (Ren et al., 2021).

The call for mental health care needs to acknowledge the diversity of theoretical models on mental health problems has been raised. Scholars from medical and non-medical disciplines, such as psychiatry, psychology, biology, neurology, philosophy, sociology, and medical history, have tried to answer questions about the essence of mental health, the cause of mental health problems, and how to classify or operationalize them. In recent decades, various theorists with differing backgrounds and traditions have debated the pros and cons of paradigms, approaches, and models (Richter & Dixon, 2023).

The young population’s prevalence of mental health challenges is on the rise. Prediction of the mental illness of students using social media gargets, especially smartphone usage and sensor data, is an intriguing research problem (Thakur & Roy, 2021). The feature variables related to daily living behavior using smartphone usage and sensor data were studied, and a model was developed using these feature variables to predict if anybody has a mental illness. An Independent-samples t-test has been used to compare the variation in means between the healthy group and the group with mental illness. Correlation analysis determined the strength of the relationship between the independent and dependent variables, and a classification model was developed to predict mental health (baseline: $n = 45$). The difference in means of various feature variables among the two groups was statistically significant ($p \leq 0.05$). Many variables are strongly correlated with various mental health predictors. The area under the curve of the prediction model for predicting stress is 82.6%, and that for depression is 74%. The results are encouraging and point towards the novel application of smartphone-based data sensing in tracking or predicting mental health issues. The study had implications for practice, such as developing a smartphone-based automated system for predicting mental health that could be a valuable tool for professionals in predicting mental health, especially in academic institutions (Thakur & Roy, 2021).

Though Laplace transform and variation of parameters methods, which are conventional methods of solving linear ODEs analytically, could be applied, the exact solutions of this kind of ODEs, which are characterized by constant coefficients, involve solving the formulated characteristic polynomial (Mat Daud, 2020; Ren et al., 2021). Since no general formula is proposed to determine the roots of a given characteristic polynomial of higher degree that is greater than 5, specifically when dealing with symbolic polynomials, mathematical software(s) are thus applied to study the behavior as even the simple simplification may seem impossible (Daud & Qing, 2021; Occhipinti et al., 2021).

(Mat Daud et al., 2019) Discussed hypertensive disorders of pregnancy as major maternal and fetal health threats worldwide, causing high levels of morbidity and mortality. The research proposed a dynamic model of hypertension during pregnancy. Differential equations governed the developed model. The stability analysis was performed using Routh-Hurwitz criteria. The research further investigated equilibrium solutions using sensitivity analysis. The method used for sensitivity analysis was normalized forward sensitivity index. The results of the sensitivity analysis of the equilibrium solution revealed the dominant parameters of the model. The parameters (factors) that need to change to minimize the future prevalence of hypertensive patients and the significance of each parameter were suggested based on the sensitivity analysis results. The results also suggested the significance of early prevention of hypertension among women of childbearing age. (Mat Daud et al., 2019).

People who have mental illness develop a variety of models to explain their conditions in the face of uncertainty. This kind of explanatory model matters since they are associated with internalized stigma and illness behaviors such as treatment preferences. Being diagnosed with a mental disorder likely raises many questions in people's minds, including whether the diagnosis is correct and, if so, what caused it. Most research on attributions for mental illness focuses on the public and the relation between their causal beliefs and expressed stigma. According to attribution theory, if the cause of mental illness is viewed as uncontrollable, the public is less likely to blame people with mental illness for their condition and think poorly of them, an argument used to support the promulgation of the biomedical model of mental illness. While subscribing to the biomedical model is associated with a lower likelihood of blaming others for having a mental illness, it tends to be positively associated with the desire for social distance, perceived dangerousness, and prognostic pessimism (Elliott, 2023).

This study seek to bridge the gap in mathematical modelling on mental illness to help understand the impact of unemployment which affect drug and substance abuse as well as mental stress and in turn show the need to use modelling in understanding mental disorders.

CHAPTER THREE

MATERIALS AND METHODS

3.1 Overview

This study developed a compartmental population based mathematical model to study the dynamics of mental illness. The model developed was based on three risk factors for mental disorders; drug or substance abuse, unemployment and mental stress. A system of eight ODEs was developed and the well-posedness discussed. The different methods of solving system of linear differential equations analytically have been established and described in various textbooks (Daud & Qing, 2021). Recent studies have indicated that numerical techniques for linear differential equations recompense acceptable results (Mutiso et al., 2022). This study performed qualitative analysis to show that the model formulated is mathematically meaningful and make biological sense. In this case solution curves are plotted to show existence and uniqueness of the suitable solutions, stability of the system and the neighborhood. Finally, through numerical simulation, the model was used to study the implications of drug abuse, unemployment and mental stress on the population dynamics of victims suffering from mental illness.

3.2 Mathematical Model Formulation

Substantial numbers of people suffer from mental illness, and its root causes are complex (Mushayabasa & Tapedzesa, 2015). It may seem hard to create a model that accounts for everything. This study suggested a S-M-P-Q-R-X-Y-Z mental illness model for compartmental population dynamics on mental illness by establishing well-thought assumptions in order to build a mathematical model that is both mathematically well-posed and physiologically useful for this study.

Let S(t) represent the susceptible humans who are eligible for working. This sub-population was assumed to have gone through the basic education system and had attained a tenable age (24 years) for either formal or informal employment. Taking the rate of recruitment to susceptible class at any given time t as λ, the study distributed S(t) class into employed and unemployed subpopulation Q(t) and P(t) respectively, where ω_1 was the portion of people who got employed joining Q(t) and $(1-\omega_1)$ transmission rate to P(t) class at any given time t. The state of employment was considered not a cause of death. However, it was a trigger to mental stress, which had an advanced impact on mental illness and would lead to heavy drug abuse. Thus, the employed subpopulation Q(t) was considered to have X(t) class, for those individuals suffering mental stress and Z(t) representing the employed sub-population abusing drugs at any given time t. Consequently, the study took M(t) to represent the unemployed subpopulation suffering from mental stress and R(t) as the unemployed sub-population abusing drugs. θ_1, θ_2 and θ_3 were transmission rates from the employed class Q(t) while τ_1, τ_2 and τ_3 transmission rates from the unemployed subpopulation P(t) to join either mental stress or drug abuse classes.

The study considered drug abuse and mental stress as the main stressors (factors) leading to mental illness. Though the state of employment has been captured, it only helps this study to identify the advanced effect it can cause on the "main stressors" leading to mental illness. Y(t) was thus taken as the total population of those individuals with mental illness at any given time t. The transmission rates to mental illness class Y(t) were α_1, ρ_1, β_1 and π_2 which were for those individuals suffering mental stress and abusing drugs regardless of the state of employment.

Consequently, mental illness does not cause death according to this study; it was however considered to trigger the causes of death, such as violent actions and suicide. This study considered that the patients suffering mental illness can recover and only die through natural death ε. Those individuals μ recovering from mental diseases were considered to have gone through proper and adequate counseling, undergone sufficient therapy, had completely healed from drug abuse, and were capable of accessing good health facilities with the right support system for patients as well as family care system and thus they cannot get mental disorders once again. The total population N(t) at any given time, t, is thus given by;

$$N(t) = S(t) + M(t) + P(t) + Q(t) + R(t) + X(t) + Y(t) + Z(t) \qquad (1)$$

where t ϵ $[0, t]$ and $t > 0$.

3.3 Assumptions of the Model

To develop a mathematical model that is suitable for our study as discussed in section 3.2 above, the following assumptions were made:

i. The model only considered a population which was eligible for working; that is, individuals who had attained the age of 24 years.

ii. The state of employment does not cause death but only trigger mental stress which can lead to mental illness or heavy drug abuse if uncontrolled.

iii. Mental illness does not cause death, it was a trigger to the causes of death such as violent actions, suicide and the patient could recover from mental illness. Mental illness Patients could die through natural death.

iv. The study considered drug abuse, unemployment and mental stress as the main causes of mental illness though drug abuse and mental stress were considered as the main stressors to mental illness.

v. The study considered people recovering from mental illness as those individuals who had gone through proper and adequate counselling, undergo sufficient therapy, had completely healed from drug abuse and were capable of accessing good health facilities with proper supportive system on patients as well as family care system and thus they cannot get mental disorders once again.

Table 3.1: Parameter description

Parameters	Description
λ	Rate of recruitment at any given time t
ρ_1	Rate of employed population Z(t) abusing drug resulting to mental illness Y(t)
ρ_2	The rate of employed population Z(t) harming drugs results in mental stress due to unhealthy work conditions.
ρ_3	The rate of employed population Z(t) abusing drugs and losing their jobs results in mental stress.
β_1	The transmission rate of unemployed sub-population with mental stress to mental illness.
β_2	The transmission rate of the unemployed subpopulation with mental stress resulting in drug abuse
θ_1	Speed of transmission of the population with a job to mental stress
θ_2	Rate of information of sub-population with a job to people abusing drugs

θ_3	Rate of losing an employment
π_1	Rate of getting mental stress as a result of drug abuse due to unemployment
π_2	Rate of getting mental illness as a result of drug abuse due to unemployment
α_1	Rate of getting mental illness as a result of mental stress caused by unhealthy work condition
α_2	Rate of abusing drugs as a result of mental stress caused by harmful work condition
ε	Rate of natural death
μ	The recovery rate of mental illness
τ_1	Rate of unemployed class join the mental stress class
τ_2	Rate of unemployed abusing drugs get mental stress
τ_3	Rate of people eligible for working get employed
ω_1	The portion of people who get a job at any given time t.
$1-\omega_1$	The number of people without an appointment at any given time

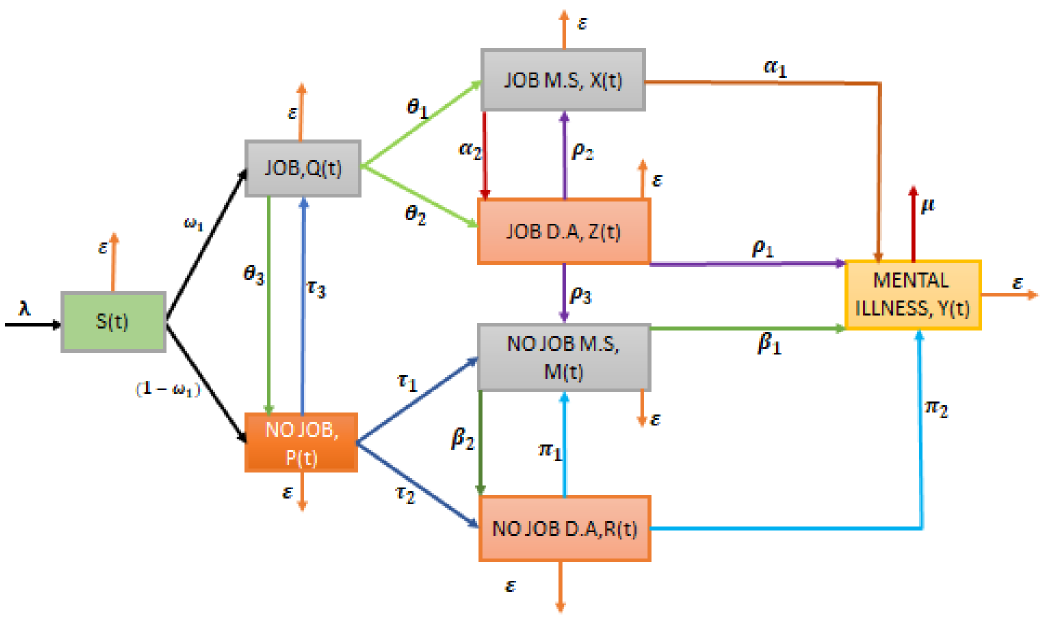

Figure 1: Mathematical model flow diagram

3.4 Modal Equation

The system of ordinary differential equations governing the S-M-P-Q-R-X-Y-Z mental illness model is given by the system of Equation (2) as;

$$
\begin{cases}
\frac{dS}{dt} = \lambda - \omega_1 S - (1-\omega_1)S - \varepsilon S \\
\frac{dM}{dt} = \tau_1 P + \pi_1 R - M(\beta_1 + \beta_2 + \varepsilon) + \rho_3 Z \\
\frac{dP}{dt} = (1-\omega_1)S + \theta_3 Q - P(\tau_1 + \tau_2 + \tau_3 + \varepsilon) \\
\frac{dQ}{dt} = \omega_1 S + \tau_3 P - Q(\theta_1 + \theta_2 + \theta_3 + \varepsilon) \\
\frac{dR}{dt} = \beta_2 M + \tau_2 P - R(\pi_1 + \pi_2 + \varepsilon) \\
\frac{dX}{dt} = \theta_1 Q - X(\alpha_1 + \alpha_2 + \varepsilon) + \rho_2 Z \\
\frac{dY}{dt} = \beta_1 M + \alpha_1 X + \pi_2 R - Y(\varepsilon + \mu) + \rho_1 Z \\
\frac{dZ}{dt} = \theta_2 Q + \alpha_2 X - Z(\rho_1 + \rho_2 + \rho_3 + \varepsilon
\end{cases}
\tag{2}
$$

3.5 Model Analysis

Mathematical analysis of the formulated model system (2) was presented in this section. The study showed the system of ordinary differential equation (2) governing the model was well-posed by proving several theorems; including the feasibility, boundedness, equilibria and stabilities of the model(Hu et al., 2021).

3.5.1 Investigating the positivity of the model

The feasibility of the model equations was investigated to ascertain if the model exists in the real domain. This was done by assessing the non-negativity of the solution and the invariant region, which described the area where the system makes both mathematical and biological sense.

3.5.2 Equilibrium Points

To study the stability of the proposed SMPQRXYZ model, the equilibrium points of the system need to be determined. It was significant to reduce the system 2 of differential equation for better model analysis. In this regard, correct substitution of parameters was done in system (2) to form system (25) on page 23 of this paper.

By Letting $\psi_1 = \beta_1 + \beta_2 + \varepsilon$, $\psi_2 = \tau_1 + \tau_2 + \tau_3 + \varepsilon$, $\psi_3 = \theta_1 + \theta_2 + \theta_3 + \varepsilon$, $\psi_4 = \pi_1 + \pi_2 + \varepsilon$, $\psi_5 = \alpha_1 + \alpha_2 + \varepsilon$, $\psi_6 = \rho_1 + \rho_2 + \rho_3 + \varepsilon$ and $\psi_7 = \varepsilon + \mu$, five equilibrium points E_0, E_1, E_2, E_3 and E_4 were determined by equating the system (25) of differential equations to zero based on different mental illness stressors considered in this study.

3.5.3 Local stability of the equilibrium points

The local stability of the five equilibria points E_0, E_1, E_2, E_3 and E_4 was determined by constructing a variation matrix and then investigating the nature of Eigen values. Using Mathematica software, the characteristic equations and their coefficient were evaluated with necessary conditions imposed. A point was said to be locally asymptomatic stable if the eigenvalues of the variation matrix were negative, otherwise unstable.

3.5.4 Global stability of drug abuse and mental stress free equilibrium points

In this study, mental stress and drug abuse were considered the main stressors of mental illness. Lyapunov function(s) were therefore proposed to study the global stability by performing analysis near the drug abuse $E_3 = (S^3, P^3, Q^3, R^3, Y^3, Z^3)$ and mental stress $E_4 = (S^4, M^4, P^4, Q^4, X^4, Y^4)$ free equilibrium points of the system.

3.6 Numerical estimation and simulation

To substantiate the analytical findings of this study, the authors integrated system (2) and studied the model numerically using the fourth order Runge-Kutta method with help of MATLAB2019a Software package. Wolfram Mathematica software was used to verify the analytical findings. Parametrization of the model by considering a secondary data from the literature review played a great role for the model validation.

CHAPTER FOUR

RESULTS AND INTEPRETATION

4.1 Overview

The analytical findings, parameter estimation and interpretation of this study are presented in this chapter. The numerical simulation of the model using data from the literature review and assumptions has been proposed. The parameters which are more influential to the model are clearly discussed and graphical representation shown.

4.2 Model Analysis.

The well-posedness of the differential equations for the model formulated in this study was demonstrated by proving several theorems on the feasibility, boundedness, equilibria as well as the local and global stabilities.

4.2.1 Positivity of the SMPQRXYZ Model

The positivity of the SMPQRXYZ Model was determined to ascertain the existence of all state variables on the real domain, $\mathbb{R}$. The following theorem was used to study the eight ODE’s described in system 2.

Theorem 1:

Let $K(t) = \{S(t), M(t), P(t), Q(t), R(t), X(t), Y(t)$ and $Z(t) \in \mathbb{R}^{8}_{+}$:

$S(0) \geq 0, M(0) \geq 0, P(0) \geq 0, Q(0) \geq 0, R(0) \geq 0, X(0) \geq 0, Y(0) \geq 0, Z(0) \geq 0\}$ then

the solution set of the modal system of equation (2) for the initial data set

$(S, M, P, Q, R, X, Y, Z)(0) \geq 0$ is positive $\forall t > 0$.

Proof:

Consider the equation on mental illness from the system of model equation (2):

$$\frac{dY}{dt} = \beta_1 M + \alpha_1 X + \pi_2 R - Y(\varepsilon + \mu) + \rho_1 Z \tag{3}$$

By inspecting, $\beta_1 M \geq 0$, $\alpha_1 X \geq 0$, $\pi_2 R \geq 0$ and $\rho_1 Z \geq 0$, and thus:

$$\frac{dY}{dt} \geq -Y(\varepsilon + \mu) \tag{4}$$

Equation (4) represents a first-order linear differential inequality and thus can be solved using the separation of variables method to obtain:

$$\int \frac{dY}{Y} \geq \int -(\varepsilon + \mu)dt \qquad (5)$$

$$In\ Y \geq -(\varepsilon + \mu)t + c$$

$$Y(t) \geq e^{-(\varepsilon+\mu)t}.e^{c} \qquad (6)$$

Since any exponent raised to a negative value (e^{-ve} is always positive) then

$$e^{-(\varepsilon+\mu)t} > 0 .$$

In the absence of mental illness and applying initial conditions, (6) become,

$$Y(t) \geq Y(0)e^{-(\varepsilon+\mu)t},\ Y(0) \geq 0,\ \ \forall t \geq 0. \qquad (7)$$

and thus $\frac{dY}{dt} = \beta_1 M + \alpha_1 X + \pi_2 R - Y(\varepsilon + \mu) + \rho_1 Z$ exist in the real domain.

Similarly, by inspecting the second equation $\frac{dS}{dt} = \lambda - \omega_1 S - (1-\omega_1)S - \varepsilon S$, for the susceptible class S(t), the value of λ is positive, that is $\lambda \geq 0$. Taking the remaining part of the inequality we have;

$$\frac{dS}{dt} \geq -\omega_1 S - (1-\omega_1)S - \varepsilon S .$$

Integrating by the separation of variables method, we have:

$$\int \frac{dS}{S} \geq \int -(1+\varepsilon)dt$$

$$In\ S \geq -(1+\varepsilon)t + c$$

$$S(t) \geq e^{-(1+\varepsilon)t}.e^{c}$$

Taking $t = 0 \Rightarrow S(0) = e^{c}$ and thus, $S(t) \geq S(0)e^{-(1+\varepsilon)t}$.

Therefore,

$\frac{dS}{dt} = \lambda - \omega_1 S - (1-\omega_1)S - \varepsilon S$ exist in the real domain, $\mathbb{R}^{+}$.

$$S(t) \geq S(0)e^{-(1+\varepsilon)t},\ S(0) \geq 0\ \ ,\forall t > 0 \qquad (8)$$

The unemployed sub-population suffering mental stress M(t) was described by those who abuse drugs and lack either formal or informal employment. To study the positivity of the ODE describing this class consider;

$$\frac{dM}{dt} = \tau_1 P + \pi_1 R - M(\beta_1 + \beta_2 + \varepsilon) + \rho_3 Z ,$$

By inspection $\tau_1 P(t) \geq 0,\ \pi_1 R(t) \geq 0$ and $\rho_3 Z(t) \geq 0$ (all are positive) and thus we proceed as follows to show if $\frac{dM}{dt} \geq -M(\beta_1 + \beta_2 + \varepsilon)$ lie of positive quadrant.

Integrating the differential equation using separation of variables, it follows that;

$$\int \frac{dM}{M} \geq \int -(\beta_1 + \beta_2 + \varepsilon) dt$$

$$In\ M \geq -(\beta_1 + \beta_2 + \varepsilon)t + C$$

Taking exponential both sides we have;

$$M(t) \geq e^{-(\beta_1+\beta_2+\varepsilon)t}.e^{c}$$

At initial, $t = 0$ and by substitution, we have $M(0) = e^{c}$ and therefore;

$$M(t) \geq M(0)e^{-(\beta_1+\beta_2+\varepsilon)t}$$

Since $M(0)e^{-(\beta_1+\beta_2+\varepsilon)t}$ is equivalent to a positive constant $(e^{-ve}$ is always positive$)$.

Then the differential equation $\frac{dM}{dt} = \tau_1 P + \pi_1 R - M(\beta_1 + \beta_2 + \varepsilon) + \rho_3 Z$, for the unemployed sub-population suffering mental stress lie on the real domain;

$$M(t) \geq M(0)e^{-(\beta_1+\beta_2+\varepsilon)t},\ M\left(0\right) \geq 0 \quad, \forall t > 0 \tag{9}$$

The ODE for the sub-population of people without a job P(t), at any given time t, is given by; $\frac{dP}{dt} = (1-\omega_1)S + \theta_3 Q - P(\tau_1 + \tau_2 + \tau_3 + \varepsilon)$. By inspecting the ODE, clearly $(1-\omega_1)S$ and $\theta_3 Q$ are positive for $\omega_1 < 1$ and thus we proceed as follows to check the positivity of the differential inequality.

Integrating by separation of variable

$$\int \frac{dP}{P} \geq \int -(\tau_1 + \tau_2 + \tau_3 + \varepsilon) dt$$

$$In\ \mathrm{P} \geq -(\tau_1 + \tau_2 + \tau_3 + \varepsilon)t + C$$

Taking exponential on both sides we have:

$$P(t) \geq e^{-(\tau_1+\tau_2+\tau_3+\varepsilon)t}.e^{c}$$

At initial, t = 0 and by substituting,

$P(0) = e^{c}$ and therefore;

$$P(t) \geq P(0)e^{-(\tau_1+\tau_2+\tau_3+\varepsilon)t}$$

The above implies that $P(0)e^{-(\tau_1+\tau_2+\tau_3+\varepsilon)t} \geq 0$ since e^{-ve} is always positive the ODE $\frac{dP}{dt} = (1-\omega_1)S + \theta_3 Q - P(\tau_1 + \tau_2 + \tau_3 + \varepsilon)$ of subpopulation of people without a job lie in real domain;

$$P(t) \geq P(0)e^{-(\tau_1+\tau_2+\tau_3+\varepsilon)t} \ , \ P(0) \geq 0 \ , \forall t > 0 \tag{10}$$

The ODE; $\frac{dQ}{dt} = \omega_1 S + \tau_3 P - Q(\theta_1 + \theta_2 + \theta_3 + \varepsilon)$ represents the sub-population of people with a job at any given time t. By inspection $\omega_1 S \geq 0$ and $\tau_3 P \geq 0$.

The differential inequality $\frac{dQ}{dt} \geq -Q(\theta_1 + \theta_2 + \theta_3 + \varepsilon)$ is then investigated to check if it lies on the real domain by integration using separation of variables method as follows;

$$\int \frac{dQ}{Q} \geq \int -(\theta_1 + \theta_2 + \theta_3 + \varepsilon)dt$$

$$In\ Q \geq -(\theta_1 + \theta_2 + \theta_3 + \varepsilon)t + c$$

$$Q(t) \geq e^{-(\theta_1+\theta_2+\theta_3+\varepsilon)t}.e^{c}$$

For $t = 0$, then $Q(0) = e^{c}$ and therefore;

$$Q(t) \geq Q(0)e^{-(\theta_1+\theta_2+\theta_3+\varepsilon)t}$$

The ODE $\frac{dQ}{dt} = \omega_1 S + \tau_3 P - Q(\theta_1 + \theta_2 + \theta_3 + \varepsilon)$ lie on the real domain, $\mathbb{R}^{+}$ since e^{-ve} is always positive.

$$Q(t) \geq Q(0)e^{-(\theta_1+\theta_2+\theta_3+\varepsilon)t} \ , \ Q(0) \geq 0, \forall t > 0 \tag{11}$$

The differential equation describing sub-population of drug abusers without an employment R(t) (unemployed) is given by $\frac{dR}{dt} = \beta_2 M + \tau_2 P - R(\pi_1 + \pi_2 + \varepsilon)$.

By inspection $\beta_2 M \geq 0, \tau_2 P \geq 0$ and thus $\frac{dR}{dt} \geq -R(\pi_1 + \pi_2 + \varepsilon)$.

By integrating using separation of variables;

$$\int \frac{dR}{R} \geq \int -(\pi_1 + \pi_2 + \varepsilon)dt$$

$$In\ R \geq -(\pi_1 + \pi_2 + \varepsilon)t + c$$

$$R(t) \geq e^{-(\pi_1+\pi_2+\varepsilon)t}.e^{c}$$

At initial $t=0,$ thus $Q(0)=e^{c}$ and therefore;

$$R(t)\geq R(0)e^{-(\pi_1+\pi_2+\varepsilon)t}$$

The differential equation $\frac{dR}{dt}=\beta_2 M+\tau_2 P-R(\pi_1+\pi_2+\varepsilon)$ thus lies on the real domain since e^{-ve} is always positive.

$$R(t)\geq R(0)e^{-(\pi_1+\pi_2+\varepsilon)t},\ R(0)\geq 0, \forall \mathrm{t}>0 \tag{12}$$

The subpopulation of employed people with a job suffering from mental stress is governed by the differential equation $\frac{dX}{dt}=\theta_1 Q-X(\alpha_1+\alpha_2+\varepsilon)+\rho_2 Z$ in which by inspection $\theta_1 Q\geq 0, \rho_2 Z\geq 0$ and $\frac{dX}{dt}\geq -X(\alpha_1+\alpha_2+\varepsilon).$

By integrating

$$\int\frac{dX}{X}\geq\int -(\alpha_1+\alpha_2+\varepsilon)dt$$

$$In\, X\geq -(\alpha_1+\alpha_2+\varepsilon)t+c$$

$$X(t)\geq e^{-(\alpha_1+\alpha_2+\varepsilon)t}.e^{c}$$

Taking $X(0)=0$ when $t=0$ and thus;

$$X(t)\geq X(0)e^{-(\alpha_1+\alpha_2+\varepsilon)t}$$

Since $X(0)e^{-(\alpha_1+\alpha_2+\varepsilon)t}\geq 0$ as $e^{-ve}>0$, then $\frac{dX}{dt}=\theta_1 Q-X(\alpha_1+\alpha_2+\varepsilon)+\rho_2 Z$ exist in a real domain;

$$X(t)\geq X(0)e^{-(\alpha_1+\alpha_2+\varepsilon)t},\ X(0)\geq 0, \forall \mathrm{t}>0 \tag{13}$$

Lastly, the differential equation governing the sub-population of those employed people abusing drugs $\frac{dZ}{dt}=\theta_2 Q+\alpha_2 X-Z(\rho_1+\rho_2+\rho_3+\varepsilon)$ is considered to inspect if it lies in the feasibility region.

By inspection, $\theta_2 Q\geq 0, \alpha_2 X\geq 0$ and thus $\frac{dZ}{dt}\geq -Z(\rho_1+\rho_2+\rho_3+\varepsilon)$.

The positivity of the differential inequality $\frac{dZ}{dt}\geq -Z(\rho_1+\rho_2+\rho_3+\varepsilon$ is inspected by performing integration as follows;

$$\int\frac{dZ}{Z}\geq\int -(\rho_1+\rho_2+\rho_3+\varepsilon)dt$$

$$In\ Z \geq -(\rho_1+\rho_2+\rho_3+\varepsilon)t + c$$

By applying exponent on both side,

$$Z(t) \geq e^{-(\rho_1+\rho_2+\rho_3+\varepsilon)t}.e^{c}$$

$Z(0) = e^{c}$ when $t = 0$ and thus;

$$Z(t) \geq Z(0)e^{-(\rho_1+\rho_2+\rho_3+\varepsilon)t}$$

Clearly, $e^{-(\rho_1+\rho_2+\rho_3+\varepsilon)t} > 0$ as $e^{-ve} > 0$ thus $\frac{dZ}{dt} = \theta_2 Q + \alpha_2 X - Z(\rho_1+\rho_2+\rho_3+\varepsilon)$ exists in $\mathbb{R}^{+}$.

$$Z(t) \geq Z(0)e^{-(\rho_1+\rho_2+\rho_3+\varepsilon)t},\ Z(0) \geq 0, \forall t > 0 \tag{14}$$

Therefore, the set solutions $S(t), M(t), P(t), Q(t), R(t), X(t), Y(t)$ and Z(t) lie in the positive quadrant $\forall t > 0$, which proves the theorem.

4.2.2: Boundedness of the solution

The consequences restricting a population's growth are vital in analogy to a dynamic population system (Agarwal & Pathak, 2014). To study the boundedness of the solution of the system around the steady states, all the state variables and parameters of the S-M-P-Q-R-X-Y-Z Mental Illness Modal are assumed to be positive $\forall t > 0$. In this regard, boundedness was determined by the following theorem:

Theorem 2:

The set

$$K = \left\{(S,M,P,Q,R,X,Y,Z): 0 \leq S+M+P+Q+R+X+Y+Z \leq \frac{\lambda}{\varepsilon}, 0 \leq K(t) \leq \frac{\lambda}{\varepsilon}\right\}$$

is a region of attraction for all solutions in the system initiating the first quadrant (positively invariant and attracts all values, $\mathbb{R}^{8}_{+}$).

Proof:

Let (S,M,P,Q,R,X,Y,Z) be any solutions with positive initial conditions $K = S+M+P+Q+R+X+Y+Z$. Computing the time derivative of K along solutions of system equation (2), we have;

$$\frac{dK}{dt} = \frac{dS}{dt} + \frac{dP}{dt} + \frac{dQ}{dt} + \frac{dR}{dt} + \frac{dM}{dt} + \frac{dX}{dt} + \frac{dY}{dt} + \frac{dZ}{dt} \tag{15}$$

Substituting model equation (2) to equation (15) we have,

$$\frac{dK}{dt} = (\lambda - \omega_1 S - (1-\omega_1)S - \varepsilon S) +$$
$$(\tau_1 P + \pi_1 R - M(\beta_1 + \beta_2 + \varepsilon) + \rho_3 Z) +$$
$$(\theta_3 Q + (1-\omega_1)S - P(\tau_1 + \tau_2 + \tau_3 + \varepsilon)) +$$
$$(\omega_1 S + \tau_3 P - Q(\theta_1 + \theta_2 + \theta_3 + \varepsilon)) + (\beta_2 M + \tau_2 P - R(\pi_1 + \pi_2 + \varepsilon)) +$$
$$(\theta_1 Q - X(\alpha_1 + \alpha_2 + \varepsilon) + \rho_2 Z) + (\beta_1 M + \alpha_1 X + \pi_2 R - Y(\varepsilon + \mu) + \rho_1 Z) +$$
$$\theta_2 Q + \alpha_2 X - Z(\rho_1 + \rho_2 + \rho_3 + \varepsilon).$$

By expanding and simplifying the above reduces to;

$$\frac{dK}{dt} = \lambda - \varepsilon S - \varepsilon M - \varepsilon P - \varepsilon Q - \varepsilon R - \varepsilon X - \varepsilon Y - \varepsilon Z - \mu Y$$

Further simplification yields;

$$\frac{dK}{dt} = \lambda - \varepsilon\left(S + M + P + Q + R + X + Y + Z\right) - \mu Y \tag{16}$$

$$\frac{dK}{dt} = \lambda - \varepsilon K - \mu Y \tag{17}$$

If there is no mental illness at any given time $t, \mu = 0$ and thus (17) reduces to

$$\frac{dK}{dt} = \lambda - \varepsilon K, \Longrightarrow K' = \lambda - \varepsilon K \Longrightarrow K' - \varepsilon K = \lambda \tag{18}$$

Using integrating factors method, we proceed as follows;

$$K'\mu - \varepsilon K\mu = \mu\lambda, \tag{19}$$

$$K'\mu - K\mu' = \mu\lambda \tag{20}$$

By product rule and taking $\mu' = \varepsilon\mu$;

$$\left(\mu K\right)' = \mu\lambda, \tag{21}$$

$$\int \frac{\mu'}{\mu} = \int \varepsilon \, dt \Longrightarrow In\,\mu = \varepsilon t + c \text{ and thus } \mu = e^{\varepsilon t}.e^{c} \tag{22}$$

Substituting we have $\left(Ke^{\varepsilon t}\right)' = e^{\varepsilon t}\lambda,$ (23)

$$\int\left(Ke^{\varepsilon t}\right)' = \int \lambda e^{\varepsilon t} dt \Longrightarrow Ke^{\varepsilon t} = \frac{1}{\varepsilon}\lambda e^{\varepsilon t} + C \text{ and thus}$$

$$K = Ce^{-\varepsilon t} + \frac{\lambda}{\varepsilon} \tag{24}$$

Since, C is a constant and as $t \Rightarrow \infty$, $\lim_{t\to\infty} K \le \frac{\lambda}{\varepsilon}$ where λ is the rate of recruitment and ε the rate of natural death then, S-M-P-Q-R-X-Y-Z Mental Illness Modal is thus bounded as

$$K = \left[(S, M, P, Q, R, X, Y, Z) : 0 \le S + M + P + Q + R + X + Y + Z \le \frac{\lambda}{\varepsilon}, 0 \le K(t) \le \frac{\lambda}{\varepsilon} \right]$$

which proves theorem 2 and therefore flow generated by the model can be considered for analysis. The S-M-P-Q-R-X-Y-Z Mental Illness Modal is thus mathematically well-posed and biologically meaningful.

4.2.3 Steady States

To study the stability of the proposed SMPQRXYZ model, the equilibrium points of the system need to be determined. In order to prove the existence of equilibrium points, the study considered several cases based on the mental illness stressors. Parameters used in the model equations were grouped as $\psi_1, \psi_2, \psi_3, \psi_4, \psi_5, \psi_6$ and ψ_7 for better organization and computations with the help of the Wolfram Mathematica Software where; $\psi_1 = \beta_1 + \beta_2 + \varepsilon$, $\psi_2 = \tau_1 + \tau_2 + \tau_3 + \varepsilon$, $\psi_3 = \theta_1 + \theta_2 + \theta_3 + \varepsilon$, $\psi_4 = \pi_1 + \pi_2 + \varepsilon$, $\psi_5 = \alpha_1 + \alpha_2 + \varepsilon$, $\psi_6 = \rho_1 + \rho_2 + \rho_3 + \varepsilon$ and $\psi_7 = \varepsilon + \mu$.

Substituting $\psi_1, \psi_2, \psi_3, \psi_4, \psi_5, \psi_6$ and ψ_7 in system (2) to form system (25) below, of differential equations which was used for the subsequent analysis.

$$\begin{cases} \frac{dS}{dt} = \lambda - \omega_1 S - (1-\omega_1)S - \varepsilon S \\ \frac{dM}{dt} = \tau_1 P + \pi_1 R - \psi_1 M + \rho_3 Z \\ \frac{dP}{dt} = (1-\omega_1)S + \theta_3 Q - \psi_2 P \\ \frac{dQ}{dt} = \omega_1 S + \tau_3 P - \psi_3 Q \\ \frac{dR}{dt} = \beta_2 M + \tau_2 P - \psi_4 R \\ \frac{dX}{dt} = \theta_1 Q - \psi_5 X + \rho_2 Z \\ \frac{dY}{dt} = \beta_1 M + \alpha_1 X + \pi_2 R - \psi_7 Y + \rho_1 Z \\ \frac{dZ}{dt} = \theta_2 Q + \alpha_2 X - \psi_6 Z \end{cases} \tag{25}$$

The resulting ODE's in the system (25) were equated to zero and solved simultaneously to determine the equilibrium point E_k, for $k \in$ □ . In all the cases, parameters are properly substituted with defined terms; a, b, c, d and e, then careful computations performed to determine the stationary points in terms of selected terms. However, it is important to note that the letters are defined and can be substituted with respective parameters as outlined in each case.

Based on different mental illness stressors considered in this study (drug abuse, unemployment and mental stress), the possible equilibrium points considered were outlined in different scenarios demonstrated below;

Case 1 (Co-existence of equilibrium point, E_0)

The existence of the first equilibrium points in this study was determined by equating system of equation (25) with zero, that is; if all stress factors of mental illness exist then; $S(t) \neq 0,\ M(t) \neq 0,\ P(t) \neq 0,\ Q(t) \neq 0,\ R(t) \neq 0,\ X(t) \neq 0,\ Y(t) \neq 0$ and $Z(t) \neq 0$. With the help of Wolfram Mathematica software, the resulting differential equations were solved by elimination method to obtain the first equilibrium point as $E_0 = (S^0, M^0, P^0, Q^0, R^0, X^0, Y^0, Z^0) = \left(s, m, p, q, r, x, y, z\right)$ where;

$$s = \frac{\lambda}{1+\grave{o}},$$

$$m = \frac{r\pi_1 + z\rho_3 + p\tau_1}{\psi_1},$$

$$p = \frac{\lambda + q\theta_3 + q\grave{o}\theta_3 - \lambda\omega_1}{\left(1+\grave{o}\right)\psi_2},$$

$$q = \frac{\lambda\left(\tau_3\left(-1+\omega_1\right) - \psi_2\omega_1\right)}{\left(1+\grave{o}\right)\left(\theta_3\tau_3 - \psi_2\psi_3\right)},$$

$$r = \frac{-z\beta_2\rho_3 - p\beta_2\tau_1 - p\tau_2\psi_1}{\pi_1\beta_2 - \psi_1\psi_4},$$

$$x = \frac{q\left(\theta_2\rho_2 + \theta_1\psi_6\right)}{-\alpha_2\rho_2 + \psi_5\psi_6},$$

$$y = \frac{-r\pi_2 - x\alpha_1 - m\beta_1 - z\rho_1}{\psi_7} \text{ and}$$

$$z = \frac{-q\alpha_2\theta_1 - q\theta_2\psi_5}{\alpha_2\rho_2 - \psi_5\psi_6}.$$

Case 2: (Unemployment free equilibrium point, E_1)

Realistically, a whole population cannot be unemployed thus some people will either have formal or informal employment at some point. To obtain unemployment free equilibrium point E_1 for this study, the study considered a case where all the people had a source of income either from formal or informal sector. If $Q(t) \neq 0$ then, $\Rightarrow$ $P(t) = 0$ and all P(t) in the system (25) is eliminated to obtain system (26) of differential equations below;

$$\begin{cases} \frac{dS}{dt} = \lambda - \omega_1 S - (1-\omega_1)S - \varepsilon S \\ \frac{dM}{dt} = \pi_1 R - \psi_1 M + \rho_3 Z \\ \frac{dQ}{dt} = \omega_1 S - \psi_3 Q \\ \frac{dR}{dt} = \beta_2 M - \psi_4 R \\ \frac{dX}{dt} = \theta_1 Q - \psi_5 X + \rho_2 Z \\ \frac{dY}{dt} = \beta_1 M + \alpha_1 X + \pi_2 R - \psi_7 Y + \rho_1 Z \\ \frac{dZ}{dt} = \theta_2 Q - \psi_6 Z + \alpha_2 X \end{cases} \tag{26}$$

The differential equations in system (26) are equated to zero and solved simultaneously using elimination method. The resulting equilibrium point is thus $E_1 = (S^1, M^1, Q^1, R^1, X^1, Y^1, Z^1) = (a_1, a_2, a_3, a_4, a_5, a_6, a_7)$ where:

$$a_1 = \frac{\lambda}{1+\grave{o}},$$

$$a_2 = \frac{a_1\omega_1}{\psi_3},$$

$$a_3 = \frac{a_2\theta_2\rho_2 + a_2\theta_1\psi_6}{-\alpha_2\rho_2 + \psi_5\psi_6},$$

$$a_4 = \frac{a_3\alpha_2 + a_2\theta_2}{\psi_6},$$

$$a_5 = \frac{a_4\beta_2\rho_3}{\pi_1\beta_2 - \psi_1\psi_4},$$

$$a_6 = \frac{-a_5\pi_1 + a_4\rho_3}{\psi_1} \text{ and}$$

$$a_7 = \frac{-a_5\pi_2 + a_3\alpha_1 + a_6\beta_1 + a_4\rho_1}{\psi_7}.$$

Case 3: (Mental illness free equilibrium point, E_2)

The model described in this study considered the state of employment, drug abuse and mental stress as the main contributors (stressors) causing mental illness Y(t). The main assumption in this case was that no individual was suffering from mental illness i.e. $Y(t)=0$. To obtain E_2**,** the study consider $Y(t)=0$, and by substitution, the resulting system (27) of ODEs was formulated;

$$\begin{cases} \frac{dS}{dt} = \lambda - \omega_1 S - (1-\omega_1)S - \varepsilon S \\ \frac{dM}{dt} = \tau_1 P + \pi_1 R - \psi_1 M + \rho_3 Z \\ \frac{dP}{dt} = (1-\omega_1)S + \theta_3 Q - \psi_2 P \\ \frac{dQ}{dt} = \omega_1 S + \tau_3 P - \psi_3 Q \\ \frac{dR}{dt} = \beta_2 M + \tau_2 P - \psi_4 R \\ \frac{dX}{dt} = \theta_1 Q - \psi_5 X + \rho_2 Z \\ \frac{dZ}{dt} = \theta_2 Q + \alpha_2 X - \psi_6 Z \end{cases} \tag{27}$$

The differential equations in model system (27) above was equated to zero and then solved simultaneously with help of Wolfram Mathematica software to get the mental illness free equilibrium point, $E_2 = (S^2, M^2, P^2, Q^2, R^2, X^2, Z^2)$ $= (b_1, b_2, b_3, b_4, b_5, b_6, b_7)$ where;

$$b_1 = \frac{\lambda}{1+\grave{o}},$$

$$b_2 = \frac{-b_1\tau_3 + b_1\tau_3\omega_1 - b_1\psi_2\omega_1}{\theta_3\tau_3 - \psi_2\psi_3},$$

$$b_3 = \frac{b_2\psi_3 - b_1\omega_1}{\tau_3}$$

$$b_4 = \frac{-b_2\alpha_2\theta_1 - b_2\theta_2\psi_5}{\alpha_2\rho_2 - \psi_5\psi_6},$$

$$b_5 = \frac{b_3\pi_1\tau_2 + b_4\rho_3\psi_4 + b_3\tau_1\psi_4}{-\pi_1\beta_2 + \psi_1\psi_4},$$

$$b_6 = \frac{b_2\theta_1 + b_4\rho_2}{\psi_5} \text{ and}$$

$$b_7 = \frac{b_6\beta_2 + b_3\tau_2}{\psi_4}.$$

Case 4: (Mental stress free equilibrium point, E_3)

In this case, the study considered a scenario where mental stress was insignificant. Suppose drug abuse is taken as the main stressor towards mental illness at any given time t. Then, the sub population of people with mental stress joining the mental illness sub population is eliminated. If their exist no mental stress patients in the society, then $X(t) = 0$, $M(t) = 0$ and mental illness result from drug abuse only i.e. $R(t) \neq 0$, $Z(t) \neq 0$. The system (26) of differential equations will thus reduce to;

$$\begin{cases} \frac{dS}{dt} = \lambda - \omega_1 S - (1-\omega_1)S - \varepsilon S \\ \frac{dP}{dt} = (1-\omega_1)S - \psi_2 P + \theta_3 Q \\ \frac{dQ}{dt} = \omega_1 S + \tau_3 P - \psi_3 Q \\ \frac{dR}{dt} = \tau_2 P - \psi_4 R \\ \frac{dY}{dt} = \pi_2 R - \psi_7 Y + \rho_1 Z \\ \frac{dZ}{dt} = \theta_2 Q - \psi_6 Z \end{cases} \quad (28)$$

Using the same approach in above cases, the resulting mental stress free-equilibrium point $E_3 = (S^3, P^3, Q^3, R^3, Y^3, Z^3)$ $= (c_1, c_2, c_3, c_4, c_5, c$ was obtained where;

$$c_1 = \frac{\lambda}{1+\grave{o}},$$

$$c_2 = \frac{c_1\psi_3 - c_1\theta_3\omega_1 - c_1\psi_3\omega_1}{\theta_3\tau_3 + \psi_2\psi_3},$$

$$c_3 = \frac{c_2\tau_3 + c_1\omega_1}{\psi_3},$$

$$c_4 = \frac{c_2\tau_1}{\psi_4},$$

$$c_5 = \frac{c_3\theta_2}{\psi_6} \text{ and}$$

$$c_6 = \frac{c_4\pi_2 + c_5\rho_1}{\psi_7}.$$

Case 5: (Drug abuse free equilibrium point, E_4)

To study effects of mental stress on sub population of those not abusing drugs, this study considered mental stress as the main stressor towards mental illness at any given time t. As a result, the sub populations of people with mental disorder will only be as a result of drug abuse and thus from the model; $M(t) \neq 0 \;\; X(t) \neq 0$ implying that $R(t) = 0 \;\; Z(t) = 0$. The resulting system (29) of differential equations was formed:

$$\begin{cases} \frac{dS}{dt} = \lambda - \omega_1 S - (1-\omega_1)S - \varepsilon S \\ \frac{dM}{dt} = \tau_1 P - \psi_1 M \\ \frac{dP}{dt} = (1-\omega_1)S - \psi_2 P + \theta_3 Q \\ \frac{dQ}{dt} = \omega_1 S + \tau_3 P - \psi_3 Q \\ \frac{dX}{dt} = \theta_1 Q - \psi_5 X \\ \frac{dY}{dt} = \beta_1 M + \alpha_1 X - \psi_7 Y \end{cases} \qquad (29)$$

Using the similar approach of elimination as in other cases and considering non abuses substance(s) for the population considered in this study then, drug abuse-free equilibrium point $E_4 = (S^4, M^4, P^4, Q^4, X^4, Y^4) \;\; = (d_1, d_2, d_3, d_4, d_5, d_6)$ was obtained where;

$$d_1 = \frac{\lambda}{1+\grave{o}},$$

$$d_2 = \frac{-d_1\tau_3 + d_1\tau_3\omega_1 - d_1\psi_2\omega_1}{\theta_3\tau_3 - \psi_2\psi_3},$$

$$d_3 = \frac{d_1 + d_2\theta_3 - d_1\omega_1}{\psi_2},$$

$$d_4 = \frac{d_3\tau_1}{\psi_1},$$

$$d_5 = \frac{d_2\theta_1}{\psi_5} \text{ and}$$

$$d_6 = \frac{d_5\alpha_1 + d_4\beta_1}{\psi_7}.$$

4.3 Stability analysis

The stability analysis of the equilibrium points was determined for the proposed model. The study considered the local stability all the equilibrium points of E_0, E_1, E_2, E_3 and E_4 constructed in section 4.2. The global stability of E_3 and E_4 is also studied to ascertain which stress factor(s) pose a greater impact on mental illness

4.3.1 Local stability

The eigenvalue method is used to study the local stability. A variation matrix is constructed, and the nature of the eigenvalues of the matrix is determined. A point is said to be locally asymptomatic stable if the eigenvalues of the variation matrix are negative, otherwise unstable.

Theorem 3:

Consider the first equilibrium point $E_0 = (S^0, M^0, P^0, Q^0, R^0, X^0, Y^0, Z^0)$ $= (s, m, p, q, r, x, y, z)$; the system is stable if $\sqrt{4\theta_3\tau_3 + \psi_2^2 - 2\psi_2\psi_3 + \psi_3^2} < -(\psi_2 + \psi_3)$ $\sqrt{4\pi_1\beta_2 + \psi_1^2 - 2\psi_1\psi_4 + \psi_4^2} < -(\psi_1 + \psi_4)$, and $\sqrt{4\alpha_2\rho_2 + \psi_5^2 + 2\psi_5\psi_6 + \psi_6^2} < -(\psi_5 - \psi_6)$ otherwise unstable.

Proof:

Let $S(t) \neq 0, M(t) \neq 0, P(t) \neq 0, Q(t) \neq 0, R(t) \neq 0, X(t) \neq 0, Y(t) \neq 0$ and $Z(t) \neq 0$.

The variation matrix is constructed by differentiating the system of equation (25) concerning (w.r.t) S, M, P, Q, R, X, Y, and Z to get *Matrix* M_0 below;

$$M_0 = \begin{pmatrix} -1-\varepsilon & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\psi_1 & \tau_1 & 0 & \pi_1 & 0 & \rho_3 & 0 \\ 1-\omega_1 & 0 & -\psi_2 & \theta_3 & 0 & 0 & 0 & 0 \\ \omega_1 & 0 & \tau_3 & -\psi_3 & 0 & 0 & 0 & 0 \\ 0 & \beta_2 & \tau_2 & 0 & -\psi_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & \theta_1 & 0 & -\psi_5 & \rho_2 & 0 \\ 0 & 0 & 0 & \theta_2 & 0 & \alpha_2 & -\psi_6 & 0 \\ 0 & \beta_1 & 0 & 0 & \pi_2 & \alpha_1 & \rho_1 & -\psi_7 \end{pmatrix}$$

Solving by the help of Wolfram Mathematica Software, the resulting eigenvalues are;

$-1-\grave{o}$,

$\frac{1}{2}\left(-\psi_2-\psi_3-\sqrt{4\theta_3\tau_3+\psi_2^2-2\psi_2\psi_3+\psi_3^2}\right)$,

$\frac{1}{2}\left(-\psi_2-\psi_3+\sqrt{4\theta_3\tau_3+\psi_2^2-2\psi_2\psi_3+\psi_3^2}\right)$,

$\frac{1}{2}\left(-\psi_1-\psi_4-\sqrt{4\pi_1\beta_2+\psi_1^2-2\psi_1\psi_4+\psi_4^2}\right)$,

$\frac{1}{2}\left(-\psi_1-\psi_4+\sqrt{4\pi_1\beta_2+\psi_1^2-2\psi_1\psi_4+\psi_4^2}\right)$,

$\frac{1}{2}\left(-\psi_5+\psi_6-\sqrt{4\alpha_2\rho_2+\psi_5^2+2\psi_5\psi_6+\psi_6^2}\right)$,

$\frac{1}{2}\left(-\psi_5+\psi_6+\sqrt{4\alpha_2\rho_2+\psi_5^2+2\psi_5\psi_6+\psi_6^2}\right)$ and $-\psi_7$.

Thus, the system $E_0=(S^0,M^0,P^0,Q^0,R^0,X^0,Y^0,Z^0)$ $=(s,m,p,q,r,x,y,z)$ is stable if:

i). $\sqrt{4\theta_3\tau_3+\psi_2^2-2\psi_2\psi_3+\psi_3^2}<-(\psi_2+\psi_3)$,

ii). $\sqrt{4\pi_1\beta_2+\psi_1^2-2\psi_1\psi_4+\psi_4^2}<-(\psi_1+\psi_4)$, and

iii). $\sqrt{4\alpha_2\rho_2+\psi_5^2+2\psi_5\psi_6+\psi_6^2}<-(\psi_5-\psi_6)$ otherwise unstable.

Theorem 4:

At the second equilibrium point $E_1=(S^1,M^1,Q^1,R^1,X^1,Y^1,Z^1)$ $=(a_1,a_2,a_3,a_4,a_5,a_6,a_7)$ discussed in case 2, the system is considered locally asymptomatic stable if $\sqrt{4\pi_1\beta_2+\psi_1^2-2\psi_1\psi_4+\psi_4^2}<-(\psi_1+\psi_4)$ and $\sqrt{4\alpha_2\rho_2+\psi_5^2-2\psi_5\psi_6+\psi_6^2}<-(\psi_5+\psi_6)$;

Otherwise, unstable.

Proof:

Let $Q(t)\neq 0$ then it implies that $P(t)=0$. Substituting this in equation 2, we have a system of equation 26. By differentiating system equation (26) described in case 2, the variation *Matrix* M_1 below is generated;

$$M_1 = \begin{pmatrix} -1-\varepsilon & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\psi_1 & 0 & \pi_1 & 0 & \rho_3 & 0 \\ \omega_1 & 0 & -\psi_3 & 0 & 0 & 0 & 0 \\ 0 & \beta_2 & 0 & -\psi_4 & 0 & 0 & 0 \\ 0 & 0 & \theta_1 & 0 & -\psi_5 & \rho_2 & 0 \\ 0 & 0 & \theta_2 & 0 & \alpha_2 & -\psi_6 & 0 \\ 0 & \beta_1 & 0 & \pi_2 & \alpha_1 & \rho_1 & -\psi_7 \end{pmatrix}$$

The resulting eigenvalues M_1 are;

$-1-\grave{o},$

$-\psi_3,$

$\frac{1}{2}\left(-\psi_1-\psi_4-\sqrt{4\pi_1\beta_2+\psi_1^2-2\psi_1\psi_4+\psi_4^2}\right),$

$\frac{1}{2}\left(-\psi_1-\psi_4+\sqrt{4\pi_1\beta_2+\psi_1^2-2\psi_1\psi_4+\psi_4^2}\right),$

$\frac{1}{2}\left(-\psi_5-\psi_6-\sqrt{4\alpha_2\rho_2+\psi_5^2-2\psi_5\psi_6+\psi_6^2}\right),$

$\frac{1}{2}\left(-\psi_5-\psi_6+\sqrt{4\alpha_2\rho_2+\psi_5^2-2\psi_5\psi_6+\psi_6^2}\right),$ and $-\psi_7$.

The system E_2 is thus stable if;

i). $\sqrt{4\pi_1\beta_2+\psi_1^2-2\psi_1\psi_4+\psi_4^2} < -(\psi_1+\psi_4)$ and,

ii). $\sqrt{4\alpha_2\rho_2+\psi_5^2-2\psi_5\psi_6+\psi_6^2} < -(\psi_5+\psi_6)$ otherwise unstable.

Theorem 5:

The system at the third equilibrium point $E_2 = (S^2, M^2, P^2, Q^2, R^2, X^2, Z^2)$ $= (b_1, b_2, b_3, b_4, b_5, b_6, b_7)$ is stable if $\sqrt{4\alpha_2\rho_2+\psi_5^2-2\psi_5\psi_6+\psi_6^2} < -(\psi_5+\psi_6)$ and $\sqrt{4\pi_1\beta_2+\psi_1^2-2\psi_1\psi_4+\psi_4^2} < -(\psi_1+\psi_4)$ otherwise unstable.

Proof:

Using the same approach used in theorem 4 above, a variation matrix M_3 is generated;

$$M_2 = \begin{pmatrix} -1-\varepsilon & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\psi_1 & 0 & \pi_1 & 0 & \rho_3 & 0 \\ 1-\omega_1 & 0 & -\psi_2 & 0 & 0 & 0 & 0 \\ 0 & \beta_2 & \tau_2 & -\psi_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\psi_5 & \rho_2 & 0 \\ 0 & 0 & 0 & 0 & \alpha_2 & -\psi_6 & 0 \\ 0 & \beta_1 & 0 & \pi_2 & \alpha_1 & \rho_1 & -\psi_7 \end{pmatrix}$$

Solving, the resulting eigenvalues are;

$-1-\grave{o}$,

$-\psi_2$,

$\frac{1}{2}\left(-\psi_1-\psi_4-\sqrt{4\pi_1\beta_2+\psi_1^2-2\psi_1\psi_4+\psi_4^2}\right)$,

$\frac{1}{2}\left(-\psi_1-\psi_4+\sqrt{4\pi_1\beta_2+\psi_1^2-2\psi_1\psi_4+\psi_4^2}\right)$,

$\frac{1}{2}(-\psi_5-\psi_6-\sqrt{4\alpha_2\rho_2+\psi_5^2-2\psi_5\psi_6+\psi_6^2}$,

$\frac{1}{2}\left(-\psi_5-\psi_6+\sqrt{4\alpha_2\rho_2+\psi_5^2-2\psi_5\psi_6+\psi_6^2}\right)$ and $-\psi_7$.

The system is stable if and $\sqrt{4\pi_1\beta_2+\psi_1^2-2\psi_1\psi_4+\psi_4^2} < -(\psi_1+\psi_4)$, Otherwise unstable.

Theorem 6:

The system $E_3 = (S^3, P^3, Q^3, R^3, Y^3, Z^3) = (c_1, c_2, c_3, c_4, c_5, c_6)$ is stable if $\sqrt{4\theta_3\tau_3+\psi_2^2-2\psi_2\psi_3+\psi_3^2} \geq 0$ and $\sqrt{4\theta_3\tau_3+\psi_2^2-2\psi_2\psi_3+\psi_3^2} < -(\psi_2+\psi_3)$ otherwise unstable.

Proof:

The variation matrix M_3 below is generated using the same approach used in the above theorem;

$$M_3 = \begin{pmatrix} -1-\varepsilon & 0 & 0 & 0 & 0 & 0 \\ 1-\omega_1 & -\psi_2 & \theta_3 & 0 & 0 & 0 \\ \omega_1 & \tau_3 & -\psi_3 & 0 & 0 & 0 \\ 0 & \tau_2 & 0 & -\psi_4 & 0 & 0 \\ 0 & 0 & \theta_2 & 0 & -\psi_6 & 0 \\ 0 & 0 & 0 & \pi_2 & \rho_1 & -\psi_7 \end{pmatrix}$$

The resulting eigenvalues are;

$-1-\grave{o}$,

$\frac{1}{2}\left(-\psi_2-\psi_3-\sqrt{4\theta_3\tau_3+\psi_2^2-2\psi_2\psi_3+\psi_3^2}\right)$,

$-\psi_4$,

$-\psi_6$, and $-\psi_7$.

The system at the equilibrium point $E_3=(S^3,P^3,Q^3,R^3,Y^3,Z^3)$ is thus stable whenever $\sqrt{4\theta_3\tau_3+\psi_2^2-2\psi_2\psi_3+\psi_3^2}\geq 0$ and $\sqrt{4\theta_3\tau_3+\psi_2^2-2\psi_2\psi_3+\psi_3^2}<-(\psi_2+\psi_3)$, otherwise unstable.

Theorem 7:

In case 5, the equilibrium point $E_4=(S^4,M^4,P^4,Q^4,X^4,Y^4)=(d_1,d_2,d_3,d_4,d_5,d_6)$ is stable if $\sqrt{4\theta_3\tau_3+\psi_2^2-2\psi_2\psi_3+\psi_3^2}\geq 0$ and $\sqrt{4\theta_3\tau_3+\psi_2^2-2\psi_2\psi_3+\psi_3^2}<-(\psi_2+\psi_3)$ otherwise unstable.

Proof:

Let's consider R(t) =0 and Z(t)=0. Substituting in the differential equation (2) system reduces the system to equation (29). Differentiating all the differential equations in the system (29) concerning S, M, P, Q, X, and Y, respectively, the variation matrix M_4 below is generated;

$$M_4 = \begin{pmatrix} -1-\varepsilon & 0 & 0 & 0 & 0 & 0 \\ 0 & -\psi_1 & \tau_1 & 0 & 0 & 0 \\ 1-\omega_1 & 0 & -\psi_2 & \theta_3 & 0 & 0 \\ \omega_1 & 0 & \tau_3 & -\psi_3 & 0 & 0 \\ 0 & 0 & 0 & \theta_1 & -\psi_5 & 0 \\ 0 & \beta_1 & 0 & 0 & \alpha_1 & -\psi_7 \end{pmatrix}$$

At this equilibrium point, the eigenvalues obtained are;

$-1-\grave{o}$,

$-\psi_1$,

$\frac{1}{2}\left(-\psi_2-\psi_3-\sqrt{4\theta_3\tau_3+\psi_2^2-2\psi_2\psi_3+\psi_3^2}\right)$,

$\frac{1}{2}\left(-\psi_2-\psi_3+\sqrt{4\theta_3\tau_3+\psi_2^2-2\psi_2\psi_3+\psi_3^2}\right)$,

$-\psi_5$, and $-\psi_7$.

The system is stable if:

i). $\sqrt{4\theta_3\tau_3+\psi_2^2-2\psi_2\psi_3+\psi_3^2}\geq 0$ and,

ii). $\sqrt{4\theta_3\tau_3+\psi_2^2-2\psi_2\psi_3+\psi_3^2}<-(\psi_2+\psi_3)$ otherwise unstable.

4.3.2: Global Stability

The Lyapunov function in theorem 8 and 9 below were proposed to study the global stability near the mental stress free equilibrium points and drug abuse free equilibrium point of the system. The impact of drug abuse and mental stress on mental illness were considered to study the what happens to the system when perturbed.

Theorem 8:

Let $K_1=\frac{1}{2}\left(S^2+P^2+Q^2+R^2+Y^2+Z^2\right)$ the first Lyapunov function for the system correspond to the equilibrium point $E_3=(S^3,P^3,Q^3,R^3,Y^3,Z^3)$ $=(c_1,c_2,c_3,c_4,c_5,c_6)$ the system is globally asymptotically stable if $\frac{dK_1}{dt}<0$ and just stable when $\frac{dK_1}{dt}=0$, otherwise unstable.

Proof

Let

$$\frac{dK_1}{dt}=S\frac{dS}{dt}+P\frac{dP}{dt}+Q\frac{dQ}{dt}+R\frac{dR}{dt}+Y\frac{dY}{dt}+Z\frac{dZ}{dt}$$

By substitution;

$$\frac{dK_1}{dt}=\left(\lambda-S-\partial S\right)+P\left(S-\omega_1 S-\psi_2 P+\theta_3 Q\right)+Q\left(\omega_1 S+\tau_3 P-\psi_3 Q\right)$$
$$+Y\left(\pi_2 R-\psi_7 Y+\rho_1 Z\right)\ +R\left(\tau_2 P-\psi_4 R\right)+Z\left(\theta_2 Q-\psi_6 Z\right)$$

Expanding and simplifying;

$$\frac{dK_1}{dt}=S\left(-S-S\partial+\lambda\right)+R\left(P\tau_2-R\psi_4\right)+Z\left(Q\theta_2-Z\psi_6\right)$$
$$+Y\left(R\pi_2+Z\rho_1-Y\psi_7\right)$$

$$\frac{dK_1}{dt}=PS-S^2-S^2\partial+S\lambda+RY\pi_2+QZ\theta_2+PQ\theta_3+YZ\rho_1+PR\tau_2$$
$$+PQ\tau_3-P^2\psi_2-Q^2\psi_3\ -R^2\psi_4-Z^2\psi_6-Y^2\psi_7-PS\omega_1+QS\omega_1$$

The system was globally asymptotically stable whenever;

$\frac{dK_1}{dt}<0$ and just stable when $\frac{dK_1}{dt}=0$,

Otherwise unstable. Numerical values were used to verify this theorem.

Theorem 9:

Let $K_2=\frac{1}{2}\left(S^2+M^2+P^2+Q^2+X^2+Y^2\right)$ be the second Lyapunov function for the linear system which corresponds to the equilibrium point $E_4=(S^4,M^4,P^4,Q^4,X^4,Y^4)=(d_1,d_2,d_3,d_4,d_5,d_6)$; the system is globally asymptotically stable if $\frac{dK_2}{dt}<0$ and just stable when $\frac{dK_2}{dt}=0$, otherwise unstable.

Proof

$$\frac{dK_2}{dt}=S\frac{dS}{dt}+M\frac{dM}{dt}+P\frac{dP}{dt}+Q\frac{dQ}{dt}+X\frac{dX}{dt}+Y\frac{dY}{dt}$$

By integrating and substituting we have;

$$=S\left(-S\left(1+\partial\right)+\lambda\right)+\ M\left(P\tau_1-M\psi_1\right)+\ P\left(S+Q\theta_3-P\psi_2-S\omega_1\right)$$
$$+Q\left(P\tau_3-Q\psi_3+S\omega_1\right)+X\left(Q\theta_1-X\psi_5\right)$$
$$+Y\left(X\alpha_1+M\beta_1-Y\psi_7\right)$$

Expanding we have:

$$\frac{dK_2}{dt}=PS\left(1-\omega_1\right)-S^2\left(1+\partial\right)+S\lambda+XY\alpha_1+MY\beta_1+QX\theta_1$$
$$+PQ\left(\theta_3+\tau_3\right)+MP\tau_1\ -M^2\psi_1-P^2\psi_2-Q^2\psi_3-X^2\psi_5-Y^2\psi_7+QS\omega_1.$$

The system was globally asymptotically stable whenever;

$$\frac{dK_2}{dt} < 0 \text{ and just stable when } \frac{dK_2}{dt} = 0 ,$$

Otherwise unstable. Numerical values were used to verify this theorem.

4.4 Numerical Simulations

In this section, the numerical analysis is done with the help of Wolfram Mathematica to substantiate the analytical findings. Values for several parameters were calculated based on the literature study, and these estimates are cited in Table 2. The study relied heavily on previously published works related to this study due to difficulty and unavailability of data on mental illness cases. The initial values of the variables used to generate graphs in figure 2-9 are; S(0)=490,500, M(0)=150,000, P(0)=120,000, Q(0)=260,000, R(0)=95,500, X(0)=250,000, Y(0)=80,000 and Z(0)=32,000. Further, we extracted more data from the 2019 national census statistical data in Kenya.

From the numerical analysis, we found that one equilibrium point E_0 was unstable and all other four equilibrium points E_1, E_2, E_3 and E_4 were stable and since our model lie in the first quadrant, we will not verify the imaginary equilibrium point. For *Theorem 3*, the eigenvalues corresponding to the equilibrium point $E_0 = (S^0, M^0, P^0, Q^0, R^0, X^0, Y^0, Z^0)$ are; -1.6, -491897, -218180, -88006.8, -15654.3,-331609, 335605, and -0.605, thus the system is unstable.

Similarly, the eigenvalues corresponding to E_1, E_2, E_3 and E_4 are;

[-1.6, -380921, -88006.8, -15654.3, -469103, -50376.6, -0.605],

[-1.6, -329157, -88006.8, -15654.3, -469103, -50376.6, -0.605],

[-1.6, -491897, -218180, -75050.6, -261738, -0.605] and

[-1.6, -2861 0, -491897, -218180, -257742, -0.605] respectively.

Inspecting using the Eigenvalue technique, the four equilibrium points are locally asymptotically stable.

Table 4.1: Parameter and value of the model

Parameter	Value	Source
λ	418,184	Kenya national bureau of Statistics(KNBS)
ω_1	291,892	(Kwobah et al., 2017)
θ_1	257,741	(Kiburi et al., 2018; Kwobah et al., 2017)
θ_2	34,152	(Kwobah et al., 2017)
θ_3	89027	Assumed
τ_1	14,776	(Kiburi et al., 2018; Kwobah et al., 2017)
τ_2	111,515	(Kwobah et al., 2017; Mutiso et al., 2022)
τ_3	202,865	(Kwobah et al., 2017), (Wanjiru et al., 2022)
α_1	27,321	(Jenkins et al., 2015)
α_2	230420	Assumed
π_1	45833.96	Assumed
π_2	29,216	(Smith et al., 2020), (Wanjiru et al., 2022)
μ	0.005	(Agarwal & Pathak, 2014)
ε	0.6	(Agarwal & Pathak, 2014)
ρ_1	67,528	(Smith et al., 2020)
ρ_2	190213	Assumed
ρ_3	3996	Assumed
β_1	11,820	(Jenkins et al., 2015)
β_2	16790	Assumed

Using the MATLABR2019a Software package, the authors numerically investigated the dynamical behavior of the model linear ODEs in the system (2) in order to corroborate the results of the analytic part of the research (Agarwal & Pathak, 2014; Mutua et al., 2022).

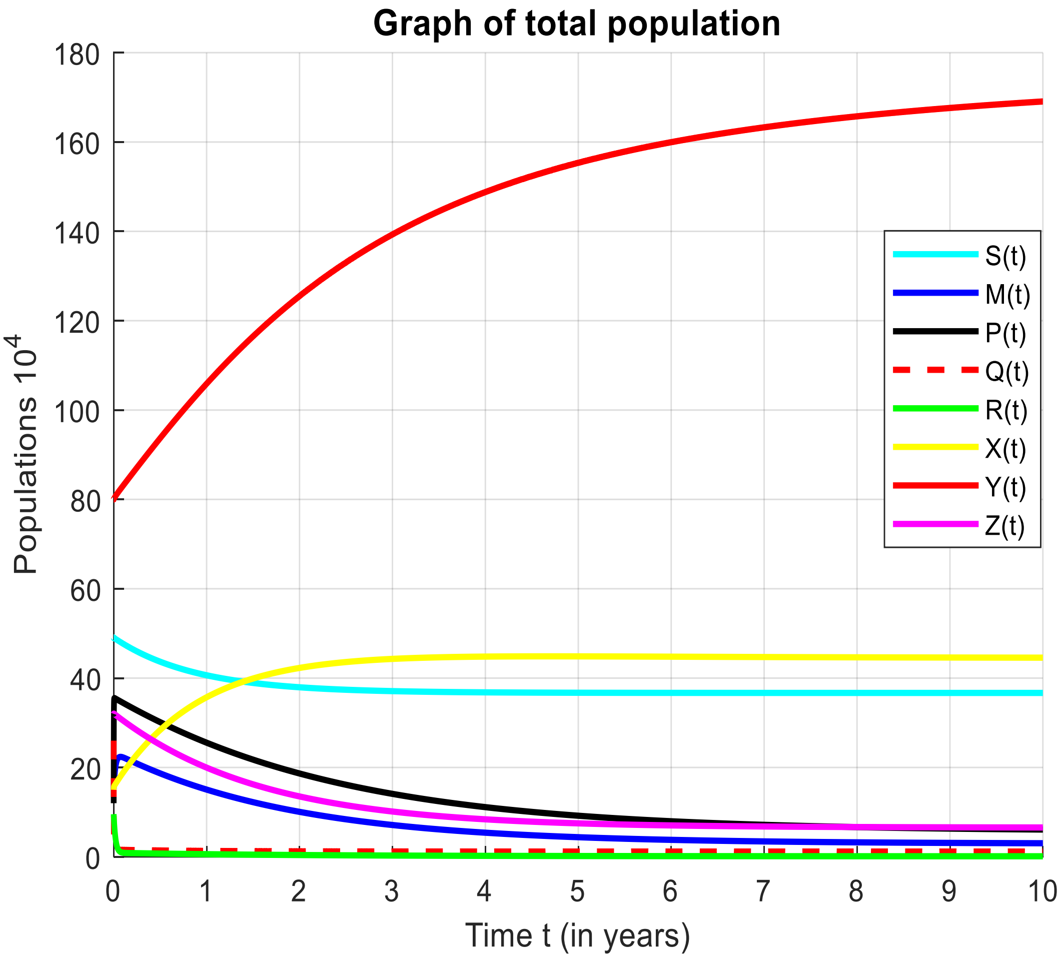

Figure 2: Dynamics of total population with respect to time, t

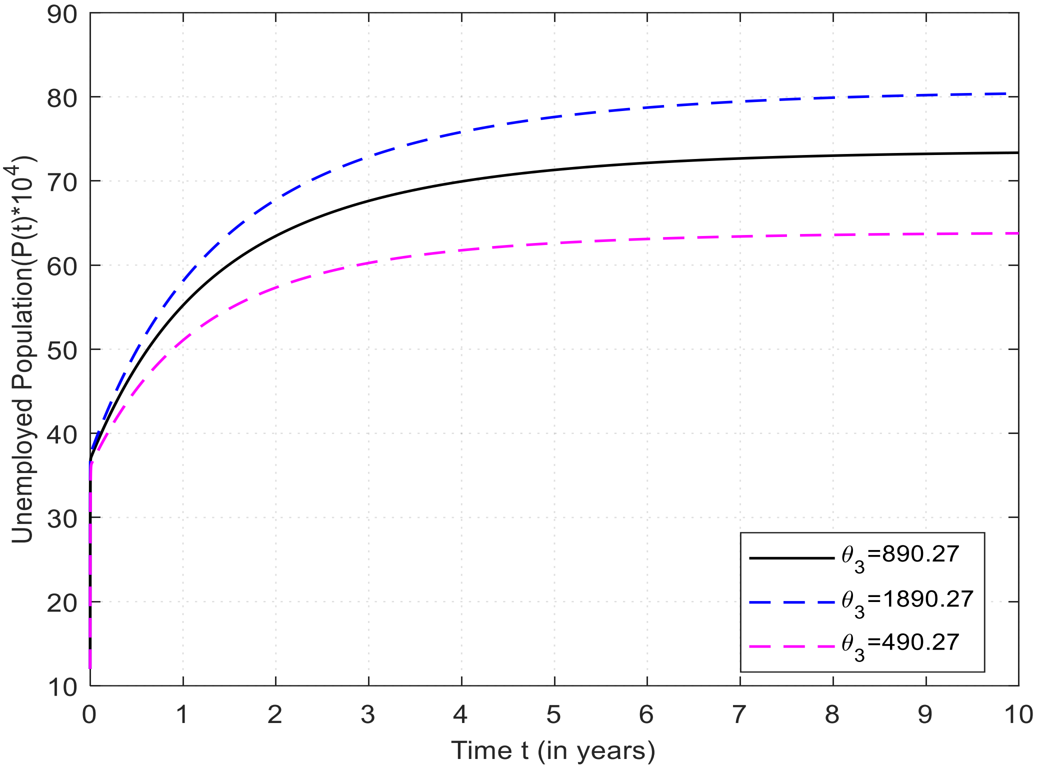

Figure 3: Dynamics of unemployed population P(t) with respect to θ_3.

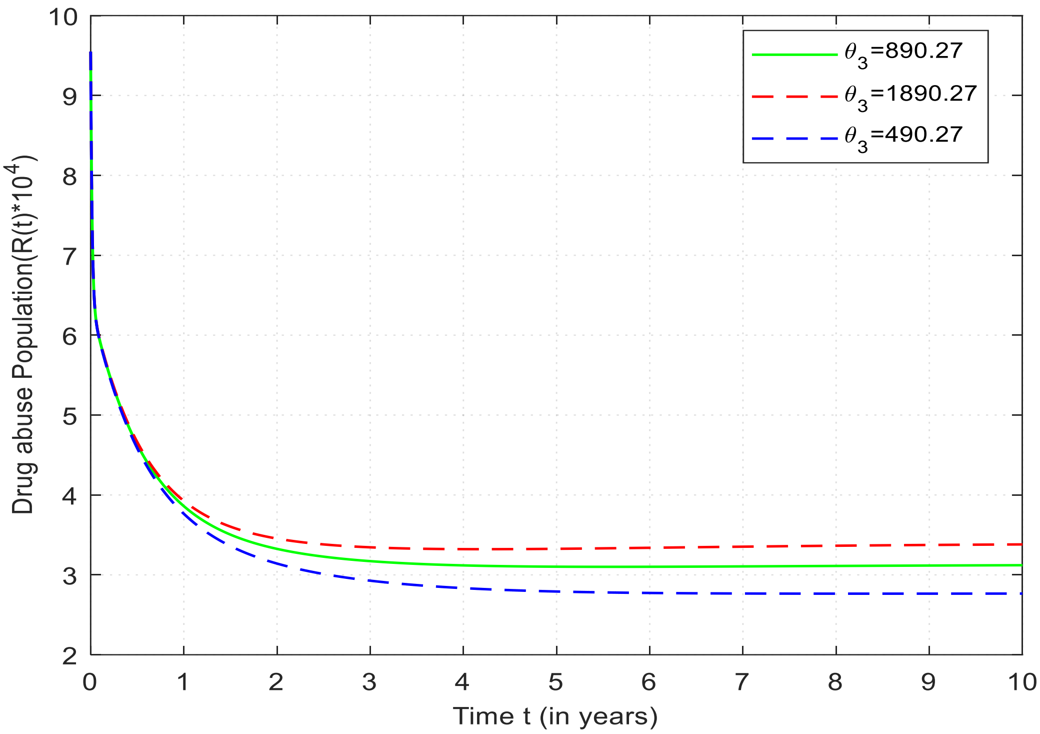

Figure 4: Dynamics of substance abuse with respect to θ_3

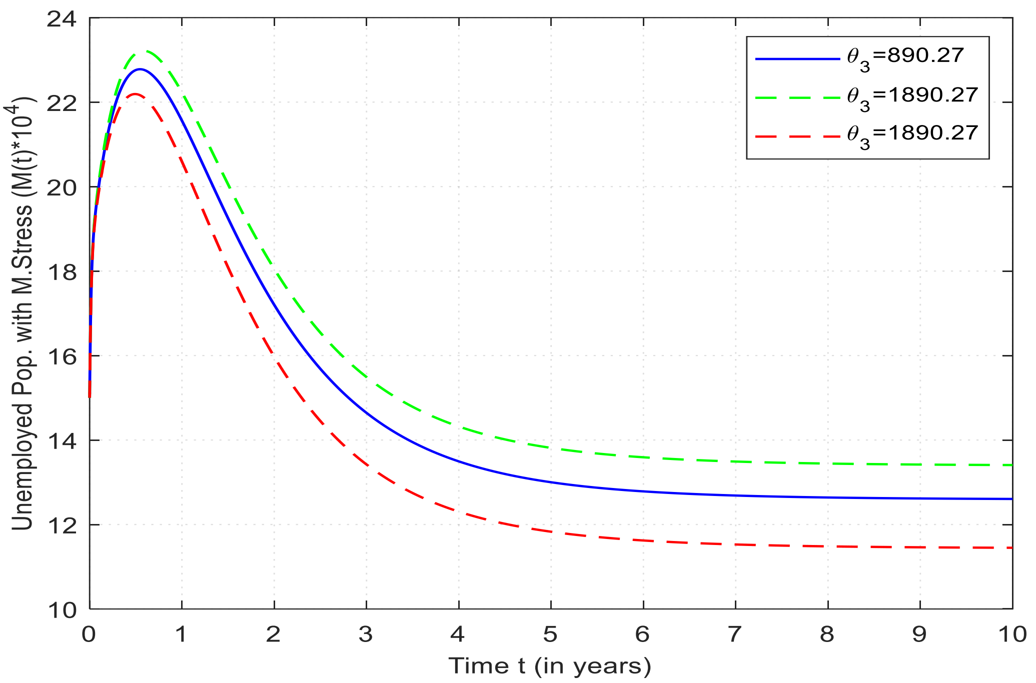

Figure 5: Effects of unemployed population with mental stress M(t) with respect to θ_3 over time

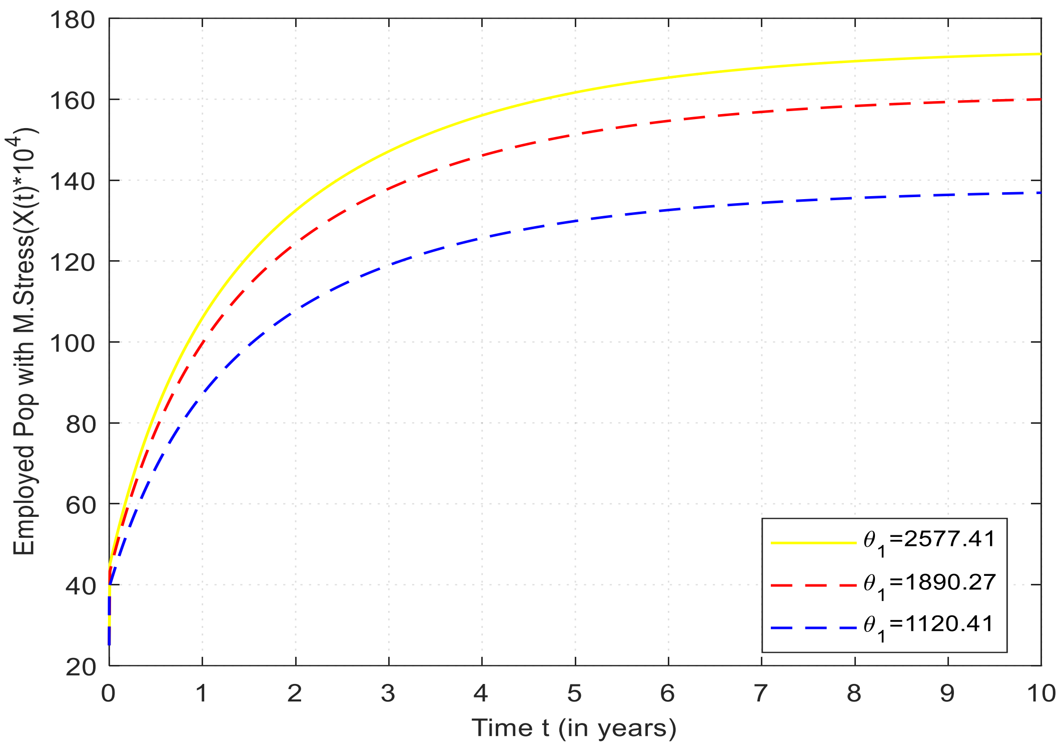

Figure 6: Effects of θ_1 on X(t) class with respect to time.

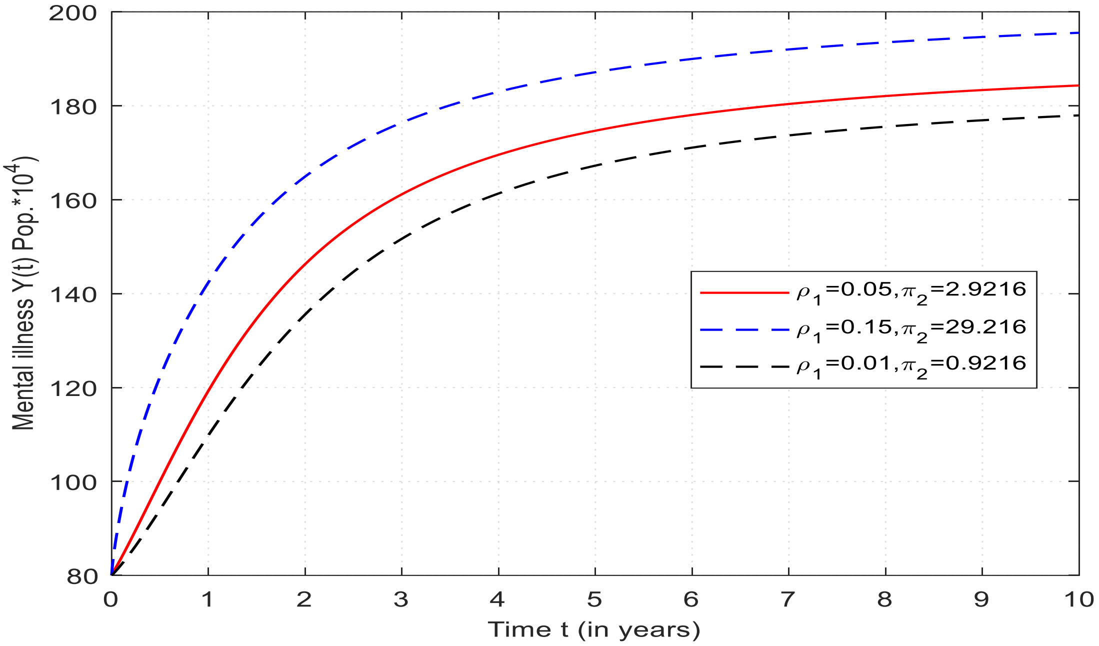

Figure 7: Effects of mental stress on mental illness Y (t) with respect to time

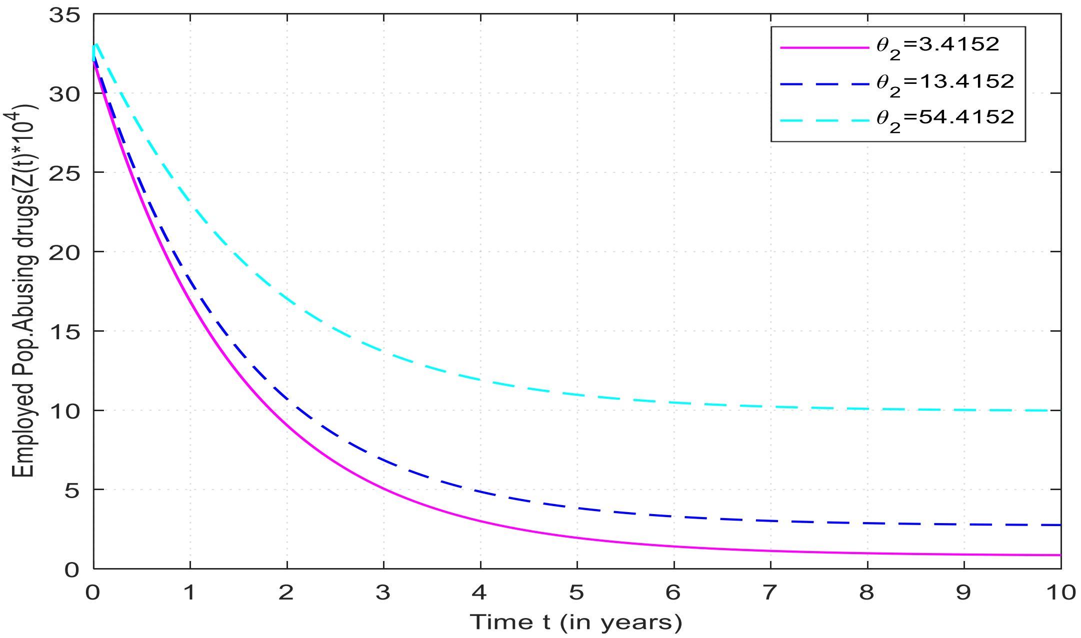

Figure 8: Effects of θ_2 on Z(t) class with respect to time.

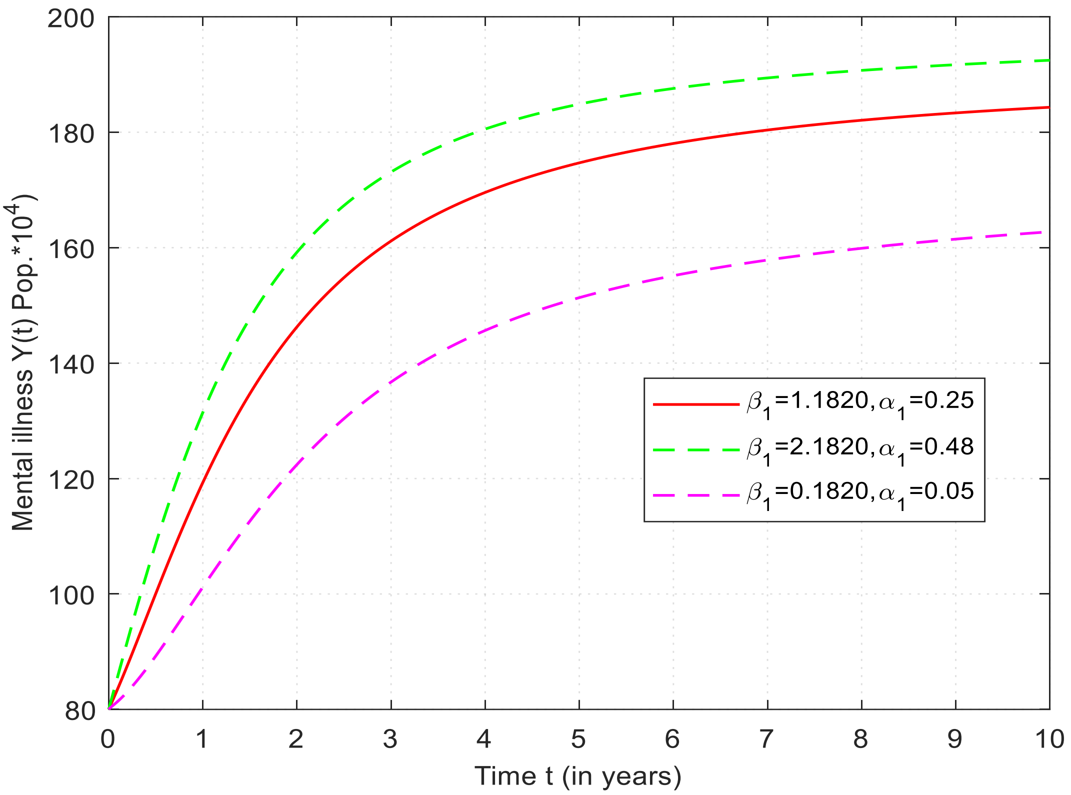

Figure 9: Effects of substance (drug) abuse on Y(t) with respect to time.

CHAPTER FIVE

DISCUSSION, CONCLUSION AND RECOMMENDATION

5.1 Discussion of the Results

In this study, state of employment, substance abuse and mental stress are selected as the main factors triggering mental illnesses. Figure 2 describes the dynamic simulation of the total population with respect to time for our modal using the original parameters cited in Table 2. The population of those suffering mental illness *Y(t)* and the employed population with mental stress *X(t)* increased exponentially until the stationary point was attained at the top end. The unemployed population *P(t)* decreased significantly due to new recruitment from the susceptible class *S(t)*. It was noted that the subpopulation of those abusing drugs, regardless of the state of employment *R(t)* and Z(t), decreased gradually until to a point where equilibrium point E_3 was attained. The subpopulations *M(t), P(t),* and *Z(t)* was observed to increase sharply to a maximum point and then decrease slowly to attain their equilibrium points at the lowest point. At some point, the subpopulations R(t) and Q(t) were observed to decrease steadily, attaining their equilibrium point and remaining constant as time t increases.

From figure 3, the unemployed population increases as the rate of losing a job (θ_3) increases from 890.27 to 1890.27. There was a significant decrease as the value of (θ_3) is reduced further to 490.27. By varying the rate of losing a job (θ_3) on the drug abuse class R(t), as shown in figure 4, this study noticed similar effects. The population of those abusing drugs increased significantly as the value of (θ_3) increased and vice versa. Unemployed population was found to be more susceptible to drugs (substance) abuse due to idleness, peer pressure, and criminal cases.

To study the effects of unemployment on mental stress, the study varied the value of (θ_3), which directly affected the number of those with a source of income, as shown in figure 5 below. There was a sharp increase in the unemployed population with mental stress M(t) as time t increased, but this trend was seen to attain a maximum point and then dropping significantly with time t. Moreover, a slight change in (θ_3) showed the similar trend with numbers in M(t) class vary proportionally as (θ_3)

increased. This indicated an increase in the mentally stressed population as more people lost a source of income.

Mental stress for employed population X(t) increased with time t, as shown in figure 6 at $\theta_1 = 2577.41$. When θ_1 was decreased from 2577.41 to 1890.41 and consequently, to 1120.41, a significant decrease in M(t) was noted. As the rate of the employed population (θ_2) increased, the graph flattened, indicating that, as more people get employed, those abusing drugs reduce significantly over time t. This indicates that if job opportunities are created, many people are engaged, reducing the chances of abusing drugs, leading to reduced cases of mental illness and stable families.

From figure 7, the study investigated substance (drug) abuse's impact on mental illness class Y(t). The population of those who had mental illness increased as the rate of the state of employment increases (regardless of whether an individual is employed or not employed). By varying the value of the rate of the working class abusing drugs(ρ_1)and also the unemployed population abusing drugs (π_2) on the mental illness population class, the results show a significant increase in mental illness within the population as time t increases until a maximum point where the equilibrium point is attained. An increase in (ρ_1) and (π_2) from 0.05 and 2.9216 to 0.15 and 29.216, respectively, as shown in figure 7, shows an increase in the population of individuals with mental illness. This is an indication that mental illness can be controlled by reducing the value of those abusing drugs (ρ_1) and (π_2) respectively.

Figure 9 shows the extent to which mental stress affects the state of mental illness within the population setup. From the graph, the trend indicates that mental stress harms mental illness. Varying the values of β_1 and α_1, which describe the transmission rate of people suffering mental stress in the general population regardless of the state of employment, showed that, mental illness can be controlled if β_1 and α_1 are under controlled. In this study, an increase β_1 from 1.1820 to 2.182 and α_1 from 0.25 to 0.48 shows a significant increase in the cases of mental illness within the community. Varying the same parameters downward reflected a downward trend in the cases of mental illness.

5.2. Conclusion

Based on eight differential equations (ODE's) described in system (2), an SMPQRXYZ mathematical model to study mental disease was established. Eight diverse categories are established to reflect distinct subpopulations, including the vulnerable, the employed, the jobless, drug abusers, the psychologically stressed, and the mentally sick. Employment status is often believed to be a primary determinant in substance addiction and mental health problems. Positiveness, boundedness, and local and global stability analyses of the equilibrium point were determined to establish the well-possedness of the mental disease model's equations.

The study constructed five equilibrium points of the system $E_0, E_1, E_2, E_3,$ and E_4. The local stability of the equilibrium points was studied using the eigenvalue method. The regional stability of each equilibrium point is then analyzed by constructing a variation matrix and determining the eigenvalues in each case. At the DFE point, the equilibrium point E_0 was unstable. All other equilibrium points were stable due to the existence of negative eigenvalues. Lyapunov function was used to study the global stability at the main stressors (drug abuse and mental stress-free equilibrium points). On the finding, the equilibrium points $E_1, E_2, E_3,$ and E_4 were locally asymptotically stable and globally stable.

MATLAB2019a software was used to run a numerical simulation that plotted out data on the relationship between unemployment, substance addiction, and mental illness. According to the findings in this study, an increase in the number of individuals who are unemployed either because they have lost their jobs or have never had them in the first place leads to an equally dramatic rise in the number of drug abusers and persons experiencing mental health problems. The effect of mental stress and substance addiction on mental illness was shown to spread in plots as the underlying stress factors for mental disease moved higher. This research recommends that governing bodies on mental health raise the working population's understanding of mental health issues so that fewer workers quit their jobs because of stress, hostile bosses, and other negative aspects of their workplaces. Taking a look at the staff's working environment as a whole is necessary.

The study suggests that parameters causing mental stress ($\theta_1, \tau_1, \pi_1, \rho_2, \rho_3$) and substance abuse ($\alpha_2, \tau_2, \beta_2, \theta_2, \theta_3$) within a given population set up to be controlled for their great impact on mental illness. A clear policy on employment for those graduating from formal education and improvement in the general working condition for the employees will lead to reduced cases of drug abuse and mental stress which will help reduce mental illness within the community. Doing real data fitting, Modelling, and simulating the competition of mental stress and substance abuse on mental illness using Holing type II response would be an interesting area for future studies.

5.3 Recommendation

There are not enough records for people with mental illness for several reasons. Disconcerting follow-up visit costs, wasted time in overcrowded mental health clinics, and a rise in inaccurate diagnosis especially from so-called "quacks" in the psychiatric and medical communities have made patients wary of seeing their psychiatrists for treatment updates. Classifiers identify instances as normal, addiction-free, dependence-with-conditions, or disorder-free. On the other hand, there has been a recent increase in cases of personality disorders, mental stress, and drug misuse.

While the government of Kenya has made some attempts to improve mental health via the ministry of health (MOH), as seen by the 'KENYA MENTAL HEALTH ACTION PLAN 2021-2025(KMHAP)', researchers still lack access to adequate data on mental health. As one of the LMICs, Kenya's Ministry of Health (MOH) is urged by this study to make its mental health data publicly available without compromising patients' right to privacy, to fully implement the policies outlined in the Kenya Mental Health Action Plan (KMHAP) for the years 2021 through 2025, and to make available sufficient capitation to fund researchers engaged in mathematical and biological modeling. This will be a nice way to inspire more people to study mental health in order to provide a precise mathematical forecast of future trends, which can then be used to inform improved mental health policies that will help the community as a whole thrive.

Mathematical modeling and simulation of drug addiction and mental stress on population dynamics of mental disease need more study in a number of areas. In the first place, we need further study into the connection between substance abuse and the development, prevalence, and intensity of mental diseases. Stress on the mind may have an effect on the beginning, development, and severity of mental disease, hence studies examining this connection are necessary.

Second, there is a need for further study into the population dynamics of mental disease, especially with regards to the manifestation and variation of mental illnesses among various demographic groups and geographic locations. Population-level variables connected to the frequency and severity of mental illness might be determined by studying the prevalence and severity of mental illnesses in various geographical locations and demographic groups.

Lastly, greater study is required to enhance current models of mental disease and investigate new models that more accurately portray the intricacies of mental health issue etiology, clinical presentation, and treatment. To better understand the interaction between many variables affecting mental health at the individual, group, or regional levels, such models might combine elements of epidemiology, system dynamics, and population-level dynamics.

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