UNIVERSITY OF EMBU

2019/ 2020 ACADEMIC YEAR

FIRST SEMESTER EXAMINATIONS

FOURTH YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN STATISTICS

STA 421: OPERATION RESEARCH III

DATE: JANUARY 14, 2020 **TIME: 11.00 – 1.00 PM**

INSTRUCTIONS:

Answer Question ONE and ANY Other TWO Questions.

QUESTION ONE (30 MARKS)

a) State four elements of a queueing system. (4 marks)

b) Briefly explain three types of sudden failure. (3 marks)

c) Explain the following terms as used in game theory; (3 marks)

i) Player

ii) Payoffs

iii) Strategies

d) Solve the given LP model by using dynamic programming technique

$\max Z = a = 9b$

subject to

$$2a + 1b \leq 25$$
$$0a + 1b \leq 11$$
$$a, b \geq 0$$

e) Briefly explain four steps of decision theory approach. (4 marks)

f) A company is planning for its sales targets and the strategies to achieve these targets. The data in terms of three sales targets their respective utilities, various strategies and appropriate probability distribution are given in the table below;

Sales targets	50	75	100
Utility	4	7	9
	Probability	Probability	Probability
Strategies			
S_1	0.6	0.3	0.1
S_2	0.2	0.5	0.3
S_3	0.5	0.3	0.2

Determine the optimal strategy. (6 marks)

g) Explain the following time estimates as used in PERT; (3 marks)

i) Optimal time (t_o)
ii) Pessimistic (t_p)
iii) Likely time (t_l)

h) Define the following terms. (4 marks)

i) Queue length.
ii) Waiting time.
iii) Average idle time or busy time distribution.
iv) Transient state of a queue.

QUESTION TWO (20 MARKS)

a) Briefly describe the customers behavior in a queue system. (5 marks)

b) A tax consulting firm has 3 counters in its office to receive people who have problems concerning their income, wealth and sales taxes. On the average, 48 persons arrive in an 8-

hour day,. Each tax adviser spends 15 minutes on the average on an arrival. If the arrivals are Poisson distributed and the service is exponential distributed, find:

i) The average number of customers in the system. (3 marks)

ii) Average number of customers waiting to be served. (2 marks)

iii) Average time a customer spends in the system. (1 mark)

iv) Average waiting time for a customer. (1 mark)

v) The number of hours each week a tax adviser spends performing his job. (2 marks)

vi) The probability that a customer has to wait before he gets served. (3 marks)

vii) The expected number of idle time tax advisers at any specified time. (3 marks)

QUESTION THREE (20 MARKS)

a) A fleet owner finds his past records that the cost per year of running a truck and resale values whose purchase price is KShs. 6000 as given below;

Year	1	2	3	4	5	6	7	8
Running cost in Shs.	1000	1200	1400	1800	2300	2800	3400	4000
Resale value in Shs	3000	1500	750	375	200	200	200	200

i) At what stage is the replacement due. (4 marks)

ii) At what value should the replacement be done. (1 mark)

b) A small project is composed of 7 activities whose time estimates are listed in the table below

Activities		Time in weeks		
i	j	t_o	t_l	t
1	2	1	1	7
1	3	1	4	7
1	4	2	2	8
2	5	1	1	1
3	5	2	5	14
4	6	2	5	8
5	6	3	6	15

i) Draw the network. (7 marks)

ii) Calculate the expected variance for each. (2 marks)

iii) Find the expected project completion time . (1 mark)

iv) Calculate the probability that the project will be completed at least 3 weeks earlier than expected. (2 marks)

v) If the project due date is 18 weeks, what is the probability of not meeting the due date. (3 marks)

QUESTION FOUR (20 MARKS)

a) Briefly explain the characteristics of a game. (5 marks)

b) Solve the game in the payoff matrix given below using linear programming method. (15 marks)

		B		
		1	2	3
A	1	3	-4	2
	2	1	-3	-7
	3	-2	4	7

QUESTION FIVE (20 MARKS)

a) Use dynamic programming to show that

$$Z = p_1 \log p_1 + p_2 \log p_2 + \ldots + p_n \log p_n$$

subject to

$$p_1 + p_2 + \ldots + p_n$$

$$\text{and} \ \ p_j \geq 0 \left(j = 1, 2, \ldots n \right)$$

Where $p_1 = p_2 = \ldots = p_n = \frac{1}{n}$, is minimum. (10 marks)

b) The owner of a chain grocery store has purchased six crates of fresh strawberries. The estimated probability distribution of potential sales of the strawberries before spoilage differs among the four stores. The following table gives the estimated total expected profit at each store, when it is allocated various numbers of crates.

Number of crates	1	2	3	4
0	0	0	0	0
1	4	2	6	2
2	6	4	8	3
3	7	6	8	4
4	7	8	8	4
5	7	9	8	4
6	1	10	8	4

For administrative reasons, the owner does not wish to split crates between stores. However, he is willing to distribute zero crates to any of his stores. Determine the maximum profit earned for this chain. (10 marks)

-END-