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Modeling Influence Tribal Coalitions in Kenya Presidential Politics: A Case Study of Kikuyu-Kalenjin versus Luo-Kamba

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Author's contribution

The sole author designed, analyzed and interpreted and prepared the manuscript.

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Abstract

A deterministic model was developed to describe the two dominant tribal coalition based voting bloc (A and B) and other tribes (C). The first order nonlinear ordinary differential equations were deduced using predator-prey equations. The system was established to lie in feasible region. The coalition free steady state was determined. The conditions necessary for local stabilities of steady states were determined using Routh-Hurwitz criteria for stability. The condition necessary for global stability of steady state were determined using Lyapunov function. The estimated numerical bound of the registered voters was obtained as 27871013. Numerical simulation was carried out using 2013 general election scenario.

Keywords: Predator-prey; Routh-Hurwitz; Lyapunov; bound and simulation.

1 Introduction

The research studies [1-4], point to the fact that 1992 and 1997 Kenyan presidential elections were largely tribal, sub tribal, clan and family based politics. According to the study [5], the 2014 estimates of the 'big' five tribes in terms of population are; Kikuyu 22%, Luhya 14%, Luo 13%, Kalenjin 12% and Kamba 11%.

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Since independence, Kenyan politics have largely been dominated by these 'big' five tribes which constitute about 72% of Kenya population. Since introduction of multiparty politics in Kenya, the presidential elections in Kenya have always been followed by election petition.

In 1992 and 1997, a Kalenjin candidate in KANU party won the two presidential elections by teaming up with Mijikenda, Maasai and some proportion of Luhya [4]. In 2002, a Kalenjin dominated KANU party had a kikuyu candidate which had hoped to retain power by having a head start of combined numerical strength of 34% (Kikuyu and Kalenjin). The opposition was very desperate to dislodge KANU Party from power in 2002, they cobbled a coalition Luo, Luhya and Kamba (combined strength 38%) under NARC party with a Kikuyu candidate. The opposition strategy of fielding a Kikuyu candidate in NARC party was to split the populous Kikuyu votes hence reducing KANU party strength, NARC party won the election [4].

In 2007, PNU party with a kikuyu candidate banking on about 28% GEMA (Kikuyu, Embu and Ameru) votes cobbled a coalition with small proportion of other tribes while ODM with a Luo candidate cobbled a coalition of Luo, Luhya and Kalenjin with combined strength of 39%. The statistics available in appendix indicate that a PNU candidate managed to split Luhya votes and with about 15% Kalenjin votes, a PNU party candidate won the hotly contested election [4].

The new Constitution was promulgated on 27 August 2010.It provides that President elected must garner half plus one of votes cast, together with a quarter of votes in at least 50% of the 47 Counties [6]. Since there no single community in Kenya with 50% of population, this only indirectly legitimized tribal coalition politics.

In 2013, Jubilee coalition with a Kikuyu candidate cobbled a coalition of Kikuyu and Kalenjin with numerical strength of about 34% while CORD party with a Luo candidate cobbled a coalition of Luo and Kamba with numerical strength of about 24%, Jubilee coalition candidate Uhuru won the election with 50.07% against his closest contestant Raila Odinga of CORD with 43.31% [7].

Worldwide, various mathematic models describing dynamics of politics has been developed:

> The research study [8], developed an ODE model describing the voting trends along two dominant political parties, which are Democratic or Republican party. The impact of interactions between apathetic individuals and potential voter in politically-charged presidential campaign was investigated. The stability of the equilibrium points was used to understand the dynamics of Democratic and Republican Party. Simulation was carried out to confirm the analytical results.
>
> The research study [9], formulated a deterministic model to describe the dynamics of two political parties. Analysis of steady states was used to understand the dynamics of the two political parties. The analytical results were confirmed by were confirmed by simulation.
>
> The research study [10], emphasized the importance of mathematical modeling in modernizing political science especially predictive purpose. The research study stressed the ability of models to link models and natural world.

The studies [2-4,7,11-13] emphasized ethnic, sub ethnic, clan based politics in Kenya but not using the mathematical modeling.

All past presidential elections are always contested with rigging claims which have been too costly to Kenyans and sometimes degenerating in election violence. To give insight into voting pattern and projected outcomes, this research paper developed a deterministic model to describe the dynamics of the coalition of ethnic voting blocs using predator-prey equations. The two ethnic coalition voting blocs (A and B) and the rest ethnic candidates (C) were developed using voting pattern for 2002, 2007 and 2013. The ethnic categorized in C sub class is assumed to be easily influenced by A and B blocs as evidenced by past elections. The steady states were determined and stability evaluated. Numerical simulation was carried out to predict political dynamics in Kenya.

2 Model Development

2.1 Introduction

The study formulated a deterministic model using predator-prey equations based on ethnic, sub ethnic; clan based voting pattern and voters' numerical strength.

2.2 Model description

Let D (t) be the total registered voters in Kenya in general election which is categorized into two coalitions of populous tribes in Kenya among the 'big' five. A (t) and B (t) are the two influential coalitions. All the other coalitions of tribes and/or candidates which are not classified in A(t) and B(t) are grouped in subclass $C(t)$. The average removal rate from registered voters due to lack of interest or death is given by μ, the recruitment rate for new voters after five years is given by π, θ_1 is the rate at which registered voters are transferred from subclass C(t) to A(t) after effective contacts using predator-prey competition, θ_2 is the rate at which registered voters are transferred from subclass C(t) to B(t) and γ is the rate at which registered voters are transferred from subclass A(t) to B(t) or vice versa. The rejected votes in general election is given by ω and the number registered voters who fails turn out in general election is given by ρ.

2.3 Model assumptions

The following assumptions were made when developing the model;

- This research study assumes homogeneous mixing of the registered voters in population.
- The study assumes rejected votes in general election(ω) and the proportions of voters who fail to turn out in general election (ρ) are shared equitably according to tribal population in Kenya.
- Since 2002, the presidential elections in Kenya are has been two tribal coalition competitions; this study assumes the impact of the third force(C) will always be not significant.
- Alliances of populous tribes are assumed to be very significant in this research study.

2.4 Model equations

The system of the first order ordinary differential equation is obtained as,

$$\dot{A} = \pi_1 + \theta_1 AC - \gamma AB - (\rho_1 + \omega_1 + \mu)A \tag{1}$$

$$\dot{B} = \pi_2 + \theta_2 BC + \gamma AB - (\rho_2 + \omega_2 + \mu)B \tag{2}$$

$$\dot{C} = \pi_3 - (\theta_1 A + \theta_2 B)C - (\rho_3 + \omega_3 + \mu)C \tag{3}$$

where,

$D(t) = A(t) + B(t) + C(t), = 1, \rho_1 + \rho_2 + \rho_3 = \rho$ and $\omega_1 + \omega_2 + \omega_3 = \omega$ and $\pi_1 + \pi_2 + \pi_3 = \pi$. Conditions for the system $(1) - (3)$ are: $\theta_1 \geq 0 \; or \; \theta_1 < 0, \theta_2 \geq 0 \; or \, \theta_2 < 0 \; and \; \gamma \geq 0 \; or \; \gamma < 0$. Let the starting conditions of the systems $(1) - (3)$ be expressed by; $A(0) = A_0, B(0) = B_0, C(0) = C_0$. Adding the systems of equations$(1) - (3)$, the change of total votes with time we obtain,

$$\frac{dD}{dt} = \pi - (\rho + \omega + \mu)D$$

3 Model Analysis

The study analyzed the model by stating and proving various theorems

3.1 Feasible region and bound of the model solutions

The study determined the feasible region and bound of the model solutions by stating and proving the theorem below.

Theorem 1. The region E is given by

$$E = \left\{A(t), B(t), C(t) \in R_+^3, D \le \frac{\pi}{\rho + \omega + \mu}\right\}$$

is positively invariant and attracting with respect to model system $(1) - (3)$,

Proof.

Let$\{A(t), B(t), C(t)\}$ be any solution of the system with initial conditions greater or equal to zero $\{A(0) \ge 0, B(0) \ge 0, C(0) \ge 0\}$.

From equation (1), $\dot{A} = \Omega_1\pi + \theta_1 AC - (\mu + \gamma B + \rho_1)A$. When $\gamma < 0$, clearly, $\dot{A} \ge -(\mu + \rho_1)A$. On integration, we obtain $\frac{d}{dt}[A(t)e^{\int_0^t -(\mu+\rho_1)dA}] \ge 0$. When $\gamma > 0$, $\dot{A} \ge -(\mu + \gamma B + \rho_1)A$. On integration, we obtain $\frac{d}{dt}[A(t)e^{\int_0^t -(\mu+\gamma B+\rho_1)dA}] \ge 0$. Clearly, $A(t)e^{\int_o^t -(\mu+\rho_1)dA} and\ A(t)e^{\int_o^t -(\mu+\gamma B+\rho_1)dA}$ are non-negative functions of t, thus A (t) stays positive.

The positivity of $B(t)\ and\ C(t)$ is proved along the same lines to obtain,

when $\gamma > 0, B(t) > B(0)e^{-(\rho_2+\omega_2+\mu)t} \ge 0$, and $C(t) > C(0)e^{-\left[\left(\theta_1\int_0^t A + \theta_2\int_0^t B\right)dt + (\rho_3+\omega_3+\mu)t\right]} \ge 0.$

The time derivative of the total population along its solution path is given by:

$\dot{D} = \dot{A} + \dot{B} + \dot{C} = \pi - (\rho + \omega + \mu)D$, this implies $\dot{D} + (\rho + \omega + \mu)D \le \pi$, which on integration becomes $D(t) \le \frac{\pi}{(\rho+\omega+\mu)}\left\{1 + Ze^{-(\rho+\omega+\mu)t}\right\}$, where, Z is the constant of integration. Hence, $\lim_{t\to\infty} D(t) \le \frac{\pi}{(\rho+\omega+\mu)}$. This proves the bounded of the solutions inside E. This implies that all the solutions of our system$(1) - (3)$, starting in E and remains in E for all $t \ge 0$. Thus E is positively invariant and attracting, and hence it is sufficient to consider the dynamics of our system in E. This completes the proof.

3.2 Coalition free steady state($\boldsymbol{E^0}$)

If the presidential elections in Kenya are not determined by two pair of dominant tribe's coalition, we equate the equations$(1) - (3)$ to zero and then set A and B to zero. We obtain, $\pi - (\rho + \omega + \mu)C = 0$. Since $\pi_1 = \pi_2 = \rho_1 = \rho_2 = \omega_1 = \omega_2 = 0$, this indicates that

$\pi_3 = \pi, \rho_3 = \rho, \omega_3 = \omega$. The steady states (E^0) is obtained as, $E^0 = (A^0, B^0, C^0) = \left(0, 0, \frac{\pi}{(\rho+\omega+\mu)}\right)$,

3.3 Existence of positive two coalition voting bloc steady state

Theorem 2.

The two coalition voting bloc positive steady state exist whenever,

$$\frac{\pi}{C^0} + B^*\gamma + \mu > A^*\gamma + (\theta_1 + \theta_2)C^* + (\rho_3 + \omega_3).$$

Proof

Let the equilibrium point be denoted by $E^* = (A^*, B^*, C^*)$. The steady state is obtained by setting system of equations $(1) - (3)$ to zero. Solving for the exact expressions A^*, B^* and C^* may not be tractable mathematically, this study propose to determine to positive values of A^*, B^* and C^* in the equations$(1) - (3)$ to obtain,

$$A^* = \frac{\pi_1}{B^*\gamma + \mu + \rho_1 + \omega_1 - \theta_1 C^*},$$

$$B^* = \frac{\pi_2}{\rho_2 + \omega_2 + \mu - (\theta_2 C^* + A^*\gamma)},$$

$$C^* = \frac{\pi_2}{\mu + \theta_1 A^* + \theta_2 B^* + \rho_3 + \omega_3} > 0.$$

Clearly, $C^* > 0$, $B^* > 0$ when $\rho_2 + \omega_2 + \mu > (\theta_2 C^* + A^*\gamma)$ and $A^* > 0$ when,

$B^*\gamma + \mu + \rho_1 + \omega_1 > \theta_1 C^*$. The expression, $B^*\gamma + \mu + \rho_1 + \omega_1 > \theta_1 C^*$, is simplified to obtain $\frac{\pi}{C^0} + B^*\gamma + \mu > A^*\gamma + (\theta_1 + \theta_2)C^* + (\rho_3 + \omega_3)$. This completes the proof.

3.4 Stability of the steady states

3.4.1 Local stability of the coalition free steady state (TFS), (E^0)

Theorem 3.

The coalition free steady state (TFS), (E^0) , is locally asymptotically stable whenever $C^0 < \frac{(\rho_1 + \omega_1 + \mu)\theta_2 + (\rho_2 + \omega_2 + \mu)\theta_1}{2\theta_1\theta_2}$ and unstable otherwise.

Proof

The Jacobi matrix of the system of equations$(1) - (3)$ is obtained as,

$$J = \begin{pmatrix} \theta_1 C - \gamma B - (\rho_1 + \omega_1 + \mu) & -\gamma A & \theta_1 A \\ \gamma B & \theta_2 C + \gamma A - (\rho_2 + \omega_2 + \mu) & \theta_2 B \\ -\theta_1 C & -\theta_2 C & -(\theta_1 A + \theta_2 B) - (\rho_3 + \omega_3 + \mu) \end{pmatrix},$$

At coalition free steady state $\{E^0 = (A^0, B^0, C^0) = \left(0, 0, \frac{\pi}{(\rho + \omega + \mu)}\right)\}$the above Jacobi matrix becomes,

$$J(E^0) = \begin{pmatrix} \theta_1 C^0 - (\rho_1 + \omega_1 + \mu) & 0 & 0 \\ 0 & \theta_2 C^0 - (\rho_2 + \omega_2 + \mu) & 0 \\ -\theta_1 C^0 & -\theta_2 C^0 & -(\rho_3 + \omega_3 + \mu) \end{pmatrix}.$$

Using Mathematica software, the eigenvalues λ_i, where $i = 1(1)3$, of the matrix $J(E^0)$ is obtained as $\lambda_1 = \theta_1 C^0 - (\rho_1 + \omega_1 + \mu), \lambda_2 = \theta_2 C^0 - (\rho_2 + \omega_2 + \mu)$ and $\lambda_3 = -(\rho_3 + \omega_3 + \mu)$. Clearly, $\lambda_3 < 0$, the condition necessary for λ_1 *and* λ_2 to be less than zero is, $C^0 < \frac{(\rho_1+\omega_1+\mu)}{\theta_1}$ and $C^0 < \frac{(\rho_2+\omega_2+\mu)}{\theta_2}$. Logically the sum of the two expressions of C^0 can be simplified as; $C^0 < \frac{(\rho_1+\omega_1+\mu)\theta_2+(\rho_2+\omega_2+\mu)\theta_1}{2\theta_1\theta_2}$. The coalition free steady state(E^0) is locally asymptotically stable whenever $C^0 < \frac{(\rho_1+\omega_1+\mu)\theta_2+(\rho_2+\omega_2+\mu)\theta_1}{2\theta_1\theta_2}$. This completes the proof. This when interpreted means that whenever the conditions are in neighbourhoods of coalition free steady state they will move towards it whenever $C^0 < \frac{(\rho_1+\omega_1+\mu)\theta_2+(\rho_2+\omega_2+\mu)\theta_1}{2\theta_1\theta_2}$ is satisfied.

3.4.2 Global stability of the coalition free steady state (TFS), (E^0)

Theorem 4.

The **coalition free steady state (TFS), (**E^0**).** is globally asymptotically stable in Lyapunov sense whenever,

$$\frac{(\pi C + \theta_1 C A^2 + \theta_2 C B^2 + \gamma A B^2)}{(\gamma B A^2 + \mu A^2 + \mu B^2 + (\theta_1 A + \theta_2 B)C^2 + (\rho + \omega + \mu)C^2)} < 1$$

,

and unstable otherwise.

Proof

We propose the Lyapunov function

$$H(A,B,C) = \frac{1}{2}(A^2 + B^2 + C^2)$$

The Lyapunov function $H(A,B,C)$ is positive definite since,

$$H(A^0,B^0,C^0) = 0 \ and \ H(A,B,C) > 0.$$

$$H(A,\dot{B},C) = A\dot{A} + B\dot{B} + C\dot{C}$$

At $E^0 = (A^0,B^0,C^0)$, $\dot{A} = \dot{B} = \dot{C} = 0$, therefore $H(A^0,\dot{B}^0,C^0) = 0$. Substituting $\dot{A}, \dot{B}$ and $\dot{C}$ from the system of equations($[2.4.1] - [2.4.3]$), we obtain

$$H(A,\dot{B},C) = A\dot{A} + B\dot{B} + C\dot{C},$$

Substituting for $\dot{A}$, $\dot{B}$ *and* $\dot{C}$ we obtain,

$$\dot{H} = A[\pi_1 + \theta_1 AC - \gamma AB - (\rho_1 + \omega_1 + \mu)A] + B[\pi_2 + \theta_2 BC + \gamma AB - (\rho_2 + \omega_2 + \mu)B] + C[\pi_3 - (\theta_1 A + \theta_2 B)C - (\rho_3 + \omega_3 + \mu)C],$$

$$\dot{H} = \pi_1 A + \pi_2 B + \pi_3 C + \theta_1 C A^2 - \gamma B A^2 - (\rho_1 + \omega_1 + \mu)A^2 + \theta_2 C B^2 + \gamma A B^2 - (\rho_2 + \omega_2 + \mu)B^2 - (\theta_1 A + \theta_2 B)C^2 - (\rho_3 + \omega_3 + \mu)C^2,$$

At coalition free equilibrium point $\pi_1 = \pi_2 = \rho_1 = \rho_2 = \omega_1 = \omega_2 = 0$, this indicates that $\pi_3 = \pi, \rho_3 = \rho, \omega_3 = \omega$,

$$\dot{H} = \pi C + \theta_1 C A^2 - \gamma B A^2 - \mu A^2 + \theta_2 C B^2 + \gamma A B^2 - \mu B^2 - (\theta_1 A + \theta_2 B)C^2 - (\rho + \omega + \mu)C^2,$$

The condition necessary and sufficient for $H(A,B,C) < 0$ is,

$$\frac{(\pi C + \theta_1 C A^2 + \theta_2 C B^2 + \gamma A B^2)}{(\gamma B A^2 + \mu A^2 + \mu B^2 + (\theta_1 A + \theta_2 B\,)C^2 + (\rho + \omega + \mu)C^2)} < 1$$

This completes the proof.

The global stability implies that the conditions of A(t), B(t) and C(t) will always move toward coalition free steady state whenever $\frac{(\pi C + \theta_1 C A^2 + \theta_2 C B^2 + \gamma A B^2)}{(\gamma B A^2 + \mu A^2 + \mu B^2 + (\theta_1 A + \theta_2 B\,)C^2 + (\rho + \omega + \mu)C^2)} < 1$,is satisfied.

3.4.3 Local stability of the of coalition bloc steady state (TFS), (E^*)

Theorem 5.

The tribe bloc steady of the system $[(2.4.1) - (2.4.4)]$ is locally asymptotically stable if the following conditions $b_2 < 0, b_3 < 0, b_4 < 0\; b_1 < 0, b_1 b_2 - b_3 > 0\; and\; b_3(b_1 b_2 - b_3) - {b_1}^2 b_4 > 0$ are satisfied and unstable otherwise.

Proof

The Jacobi matrix of the system of equations $[(2.4.1) - (2.4.3)]$ is obtained as,

$$J = \begin{pmatrix} \theta_1 C - \gamma B - (\rho_1 + \omega_1 + \mu) & -\gamma A & \theta_1 A \\ \gamma B & \theta_2 C + \gamma A - (\rho_2 + \omega_2 + \mu) & \theta_2 B \\ -\theta_1 C & -\theta_2 C & -(\theta_1 A + \theta_2 B\,) - (\rho_3 + \omega_3 + \mu) \end{pmatrix},$$

At tribe bloc steady state $\{E^* = (A^*, B^*, C^*)\}$ the above Jacobi matrix becomes,

$$J(E^*) = \begin{pmatrix} \theta_1 C^* - \gamma B^* - (\rho_1 + \omega_1 + \mu) & -\gamma A^* & \theta_1 A^* \\ \gamma B^* & \theta_2 C^* + \gamma A^* - (\rho_2 + \omega_2 + \mu) & \theta_2 B^* \\ -\theta_1 C^* & -\theta_2 C^* & -(\theta_1 A^* + \theta_2 B^*\,) - (\rho_3 + \omega_3 + \mu) \end{pmatrix},$$

Using the Mathematics software, the characteristic polynomial of the above matrix $[\,J(E^*)]$ is obtained as, $b_1\lambda^3 + b_2\lambda^2 + b_3\lambda + b_4 = 0$, where $\lambda_i, i = 1(1)3$ are eigenvalues and

$$b_1 = -1 < 0,$$

$$\begin{aligned} b_2 &= A^*\gamma + C^*\theta_1 + C^*\theta_2 - (B^*\theta_2 + B^*\gamma + 3\mu + A^*\theta_1 + \rho_1 + \rho_2 + \rho_1 + \omega_1 + \omega_2 + \omega_3), \\ b_3 &= (2A^*\gamma - 2A^*\gamma - 3\mu)\mu + A^*\gamma\rho_1 - 2\mu\rho_1 - B^*\gamma\rho_1 - 2\mu\rho_2 - \rho_1\rho_2 + A^*\gamma\rho_3 - B^*\gamma\rho_3 - 2\mu\rho_3 \\ &\quad - \rho_1\rho_3 - \rho_2\rho_3 + A^*\gamma\omega_1 - 2\mu\omega_1 - \rho_1\omega_1 - \rho_3\omega_1 - B^*\gamma\omega_2 - 2\mu\omega_2 - \rho_1\omega_2 \\ &\quad - \rho_3\omega_2 - \omega_1\omega_2 + \left(A^*\gamma - B^*\gamma - 2\mu - \rho_1 - \rho_2 - \omega_1 - \omega_2\right)\omega_3 \\ &\quad + \theta_1\left(A^{*2}\gamma + 2C^*\mu - A^*\left((B^* + C^*)\gamma + 2\mu\right) + (A^* + B^* - C^*)C^*\theta_2 - A^*\rho_1 \right. \\ &\quad \left. - A^*\rho_2 + C^*\rho_2 + C^*\rho_3 - A^*\omega_1 - A^*\omega_2 + C^*\omega_2 + C^*\omega_3\right) \\ &\quad + \theta_2\Big(B^*(A^* + B^* - C^*)\gamma - 2(B^* - C^*)\mu + (-B^* + C^*)\rho_1 - B^*\rho_2 \\ &\quad - B^*(\omega_1 + \omega_2) + C^*\left(\rho_3 + \omega_1 + \omega_3\right)\Big), \end{aligned}$$

$$\begin{aligned}b_4 = \Big(&(A^*\gamma - B^*\gamma - \mu)\mu + (A^*\gamma - \mu)\omega_1 - \rho_2(B^*\gamma + \mu + \omega_1) + \rho_2\big(A^*\gamma - \mu - \rho_2 - \omega_2\big)\\
&- (B^*\gamma + \mu + \omega_1)\omega_2\Big)\big(\mu + \rho_3 + \omega_3\big)\\
&+ \theta_1\Big(\mu\Big(A^{*2}\gamma + C^*\mu - A^*\big((B^* + C^*)\gamma + \mu\big)\Big) - A^*B^*\gamma\rho_2 - A^*\mu\rho_2 + C^*\mu\rho_2\\
&- A^*B^*\gamma\rho_3 + C^*\mu\rho_3 + C^*\rho_2\rho_3 + A^{*2}\gamma\omega_1 - A^*\mu\omega_1 - A^*\rho_2\omega_1\\
&+ A^*\rho_1\big(A^*\gamma - \mu - \rho_2 - \omega_2\big) - A^*B^*\gamma\omega_2 - A^*\mu\omega_2 + C^*\mu\omega_2 + C^*\rho_3\omega_2\\
&- A^*\omega_1\omega_2 - C^*\big(A^*\gamma - \mu - \rho_2 - \omega_2\big)\omega_3\\
&+ C^*\theta_2\Big((A^* + B^* - C^*)\mu + A^*\big(\rho_1 + \omega_1\big) + B^*\big(\rho_2 + \omega_2\big) - C^*\big(\rho_3 + \omega_3\big)\Big)\Big)\\
&+ \theta_2\Big(\mu(B^*(A^* + B^* - C^*)\gamma + (-B^* + C^*)\mu) + B^*C^*\gamma\rho_3 + C^*\mu\rho_3 + A^*B^*\gamma\omega_1\\
&- B^*\mu\omega_1 + C^*\mu\omega_1 + C^*\rho_3\omega_1 - B^*\rho_2(B^*\gamma + \mu + \omega_1)\omega_3\\
&+ \rho_1\Big(A^*B^*\gamma + (-B^* + C^*)\mu - B^*\big(\rho_2 + \omega_2\big) + C^*\big(\rho_3 + \omega_3\big)\Big)\Big).\end{aligned}$$

The Routh table is developed in the Appendix. By Routh-Hurwitz criteria for stability, the system $[(2.4.1) - (2.4.4)]$ is locally asymptotically stable at tribe bloc steady state (TFS),(E^*) if and only if $b_2 > 0, b_3 > 0, b_4 > 0$ $b_1 > 0, b_1 b_2 - b_3 > 0$ and $b_3(b_1 b_2 - b_3) - {b_1}^2 b_4 > 0$, are satisfied and unstable otherwise.

3.4.4 Global stability of the coalition bloc steady state (TFS), (E^*)

Theorem 6.

The coalition bloc steady state (EEP),(E^*), is globally asymptotically stable in Lyapunov sense whenever,

$$\frac{A\big[\gamma A^*B^* + \big(\rho_1 + \omega_1 + \mu\big)A^* + \theta_1 AC\big] + B\big[\big(\rho_2 + \omega_2 + \mu\big)B^* + \theta_2 BC + \gamma AB\big] + C[(\theta_1 A^* + \theta_2 B^*)C^* + \big(\rho_3 + \omega_3 + \mu\big)C^*]}{A\big[\theta_1 A^*C^* + \gamma AB + \big(\rho_1 + \omega_1 + \mu\big)A\big] + B\big[\theta_2 B^*C^* + \gamma A^*B^* + \big(\rho_2 + \omega_2 + \mu\big)B\big] + C[(\theta_1 A + \theta_2 B)C + \big(\rho_3 + \omega_3 + \mu\big)C]} < 1,$$

and unstable otherwise.

Proof

We propose the Lyapunov function $F(A,B,C) = \frac{1}{2}(A^2 + B^2 + C^2)$ which is positive definite since, $F(A^*,B^*,C^*) = 0$ and $F(A,B,C) > 0$. Taking time derivative of the Lyapunov function we obtain $F(A,\dot{B},C) = A\dot{A} + B\dot{B} + C\dot{C}$. At $E^* = (A^*,B^*,C^*)$, $\dot{A} = \dot{B} = \dot{C} = 0$, therefore $F(A^*,B^*,C^*) = 0$. Substituting $\dot{A}, \dot{B} and\ \dot{C}$ from the system of equations($[2.4.1] - [2.4.4]$), we obtain

$$\begin{aligned}F(A,\dot{B},C) = A\big[\pi_1 + \theta_1 AC - \gamma AB - (\rho_1 + \omega_1 + \mu)A\big] + B\big[\pi_2 + \theta_2 BC + \gamma AB - (\rho_2 + \omega_2 + \mu)B\big]\\
+ C\big[\pi_3 - (\theta_1 A + \theta_2 B)C - (\rho_3 + \omega_3 + \mu)C\big].\end{aligned}$$

At coalition bloc steady state

$$\begin{aligned}&\pi_1 = \gamma A^*B^* + \big(\rho_1 + \omega_1 + \mu\big)A^* - \theta_1 A^*C^*, \pi_2 = \big(\rho_2 + \omega_2 + \mu\big)B^* - \theta_2 B^*C^* - \gamma A^*B^* and\\
&\pi_3 = (\theta_1 A^* + \theta_2 B^*)C^* + \big(\rho_3 + \omega_3 + \mu\big)C^*.\end{aligned}$$

$$\begin{aligned}F(A,\dot{B},C) = A\big[&\gamma A^*B^* + \big(\rho_1 + \omega_1 + \mu\big)A^* - \theta_1 A^*C^* + \theta_1 AC - \gamma AB - (\rho_1 + \omega_1 + \mu)A\big]\\
&+ B\big[\big(\rho_2 + \omega_2 + \mu\big)B^* - \theta_2 B^*C^* - \gamma A^*B^* + \theta_2 BC + \gamma AB - (\rho_2 + \omega_2 + \mu)B\big]\\
&+ C\big[(\theta_1 A^* + \theta_2 B^*)C^* + \big(\rho_3 + \omega_3 + \mu\big)C^* - (\theta_1 A + \theta_2 B)C - (\rho_3 + \omega_3 + \mu)C\big],\end{aligned}$$

The condition necessary and sufficient for $F(A,B,C) < 0$ is,

$$\frac{A[\gamma A^*B^* + (\rho_1 + \omega_1 + \mu)A^* + \theta_1 AC] + B[(\rho_2 + \omega_2 + \mu)B^* + \theta_2 BC + \gamma AB] + C[(\theta_1 A^* + \theta_2 B^*)C^* + (\rho_3 + \omega_3 + \mu)C^*]}{A[\theta_1 A^*C^* + \gamma AB + (\rho_1 + \omega_1 + \mu)A] + B[\theta_2 B^*C^* + \gamma A^*B^* + (\rho_2 + \omega_2 + \mu)B] + C[(\theta_1 A + \theta_2 B)C + (\rho_3 + \omega_3 + \mu)C]} < 1.$$

This completes the proof.

4 Numerical Results

We will estimate parameter, confirm stability of coalition free equilibrium point and simulate the model using Matlab in built ordinary differential equations solver.

4.1 Parameter estimation

According to the 2014 Kenya population estimates by the study (1), the ethnic composition in Kenya was: Kikuyu 22%, Luhya 14%, Luo 13%, Kalenjin 12%, Kamba 11%, Kisii 6%, Meru 6%, other African 15%, non-African (Asian, European, and Arab) 1%.

Our research study assumes the following while estimating the model parameters;

i. The life expectancy in Kenya 2014 was 63.52 years (1). Kenyans are eligible to pick identity cards after attaining 18 years which is a prerequisite to voting. Assuming all Kenyan citizens pick voting card at 18 years. The average voting period is 45.52 years (63.52 -18). We assume a Kenyan voter is likely to vote 9.104 times (45.52 /5) because elections occur after every five years. The average removal rate (μ) from voting bracket is 0.1098(1/9.104).
ii. A case in point, in 2013 general election TNA party whose majority supporters were Kikuyus' formed coalition with URP whose majority supporters were Kalenjins' formed Jubilee coalition party (assumed to be A), based on ethnic representation in the population as per the study (1), π_1 is assumed to be 0.34(0.22+0.12). Also ODM party whose majority supporters were Luos' formed coalition with Wiper party whose majority supporters were Kambas' formed CORD (assumed to be B), based on ethnic representation in the population as per the study (1), π_2 is assumed to be 0.24(0.13+0.11).
iii. The statistic available in the literature indicates from 1992 to 2013, the positive change of new voters is 125.2036%. From 1992 to 2013, Kenyans have participated in five general elections, this implies on average the number of voters increase by 25.0407% per general election.
iv. According to research study (2), the percentage of rejected votes was 0.88% and the percentage of voters who did not turn out was 16%. The study assumes rejected votes in general election (ω) percentage of voters who fails to turn out in general election (ρ) are shared equitably according to population. Therefore $\omega_1 = 0.002992$, $\omega_2 = 0.002112, \omega_3 = 0.003696$, $\rho_1 = 0.0544$, $\rho_2 = 0.0384$ and $\rho_3 = 0.0672$. The study further assumes the those parameters may not vary significantly in future.
v. According to the data available in study (2), 84.24% of voters in subclass C voted for candidate in subclass A and subclass B. The data further indicates candidate in A got 38.26% and candidate in B got 45.98%. The study assumes $\theta_1 A = 0.3826$ and $\theta_2 B = 0.4598$. The study further assumes an ideal case whereby individuals in subclasses A and B cannot vote against their subclass hence $\gamma = 0$.
vi. According to research study (1), the total registered voter in 2013 general election was 14352533. Our study assumes the votes were evenly distributed according to 2013 general election. Therefore, $A(0) = 6173433, B(0) = 5340546$ and $C(0) = 707074$, these were used as the initial condition of the model.

4.2 Numerical simulations

We shall simulate the first order ODE's using the Matlab in built solver.

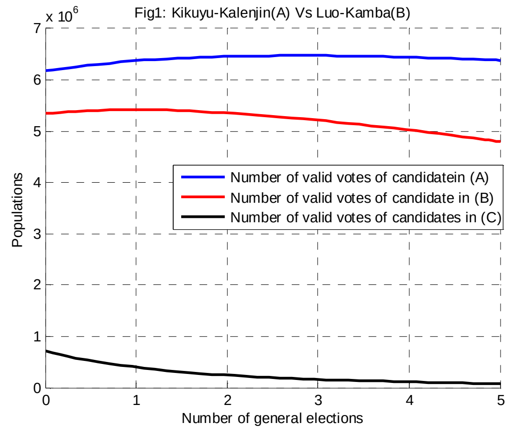

Fig. 1. The numerical simulation of the full model when the conditions of 2013 remain the same that is Jubilee coalition (Kikuyu-Kalenjin) and Cord coalition (Luo-Kamba)

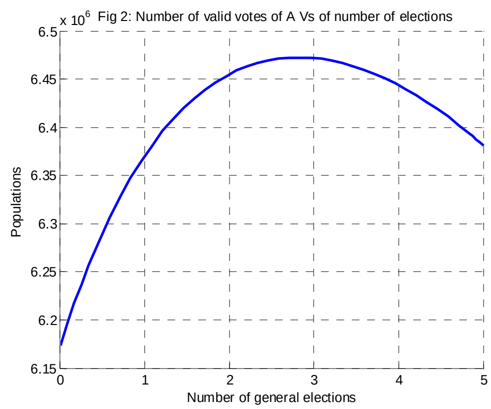

Fig. 2. If the conditions of 2013 are maintaned,the numerical simulation indicates that the number of votes of Jubilee coalition will increase up to over $6.45x10^6$ by third general election and then reduce to below $6.4x10^6$ by fifth general election

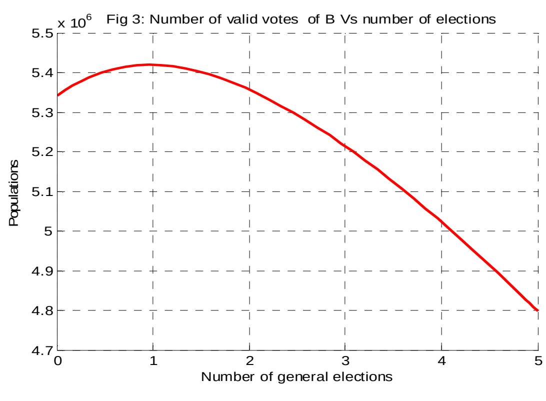

Fig. 3. If the conditions of 2013 are maintaned,the numerical simulation indicates that the number of votes of Cord coalition will increase up to over $5.4x10^6$ by first general election and then reduce to about $4.8x10^6$ by fifth general election

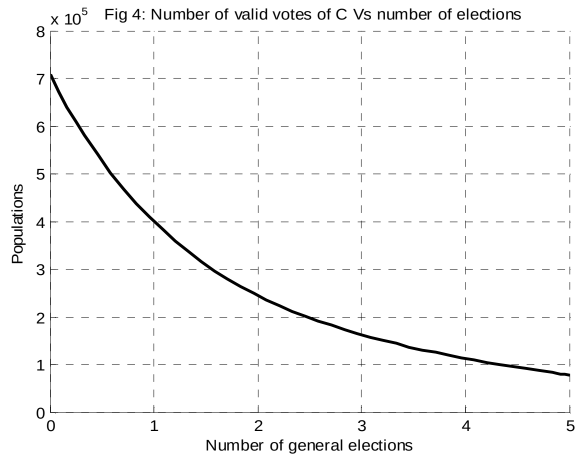

Fig. 4. If the conditions of 2013 are maintaned,the numerical simulation indicates that the number of votes in subclass will decrease up to below 1×10^6 by fifth general election

4.3 Estimated numerical results

The total registered bound voter's bound is obtained as,

$\mathrm{D(t)} \leq \frac{3060238}{0.1098+0.0088+0.0016} = 25{,}459{,}600$. Interpretation of this means that the total registered voters in future general election will always be less or equal $25{,}459{,}600$ so long as Kenya average life expectancy is around 63.52 years, average new voter recruitment in general election is about 3060238, rejected votes 0.88% and percentage of voters who fails to turn out 16%.Whenever $\mathrm{D(t)} > 25{,}459{,}600$,it is likely to reduces asymptotically to the bound.

The condition necessary for local and global stability of coalition free equilibrium point was estimated numerically using parameter in this study are obtained as $25{,}459{,}600 < 2175950$ and $0.71782\,1 < 1$ respectively. The condition necessary for local stability is false numerically hence the coalition free equilibrium is locally unstable however it is possible to attain global stability. Determining coalition blocs explicitly analytically and its stabilities numerically will be part of future research.

5 Results and Discussion

In concurrent with research studies of [8,9], this research study developed a deterministic model and deduced first order differential equations, determined equilibrium point and carried out numerical simulation using Matlab software, however unlike those research studies which used party based politics, my model was developed based on tribal coalition based politics. Kenya Presidential total registered voter data for the 2013 general election was used in this model simulation as the starting general election.

In concurrent with research articles [2-4,7,11-13], Kenya presidential politics is ethnic based, however this research study used mathematical deterministic model approach and predicted their dynamics and estimated the total voter's bound in Kenya.

This research study if disseminated well to Kenya populace is likely to reduce election tension and rigging claims because any likely tribal coalition formed can fit in the model and, if good estimates of: rejected votes, voters who fail to turn out and how coalition are likely to sway voters in rival coalitions, the election outcomes can be predicted with some degree of precision. The analysis of the other possible tribal coalitions will be part of future research.

6 Conclusion

The total registered voters are likely to be less or equal to 25,459,600 in Kenya if life expectancy, rejected votes, voter who fail to turn out in general election and recruitment of voters do not vary significantly from 2013 general election. The simulations indicate Kikuyu-Kalenjin coalition is likely to win the next three presidential if the conditions of 2013 are maintained. This model hold great promise in predicting the outcome of ethnic based presidential elections in Kenya hence reducing the burden perpetual disputes associated with outcome.

Competing Interests

Author has declared that no competing interests exist.

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Appendix

Table 1. Analysis of the presidential results 1992 to 2013 according to ethnic affiliation

Voting block/ year	Agikuyu (A)			Luo (L)	Kalenjin (K)	Rest (R)				Totals	Source
1992	Candidates	Matiba	Kibaki	Jaramogi	Moi	Tsuma	Mwau	Anyona	Nga'ng'a		(3)
	Votes	1404266	1050617	944197	1962862	15393	6449	14253	8527		
	Total votes	2454883		944197	1962862		44622			5406664	
1997	Candidates	Kibaki		Raila	Moi	Kijana	Ngilu	Others			(4)
	Votes	1895527		665725	2445801	505542	469907	Assumed Negligible			
	Total votes	1895527		665725	2445801		975449			5982502	
2002	Candidates	Kibaki	Kenyatta	-	-	Nyachae	Orengo	Ng'ethe			(4)
	Votes	3637318	1839575	-	-	380097	24547	10344			
	Total votes	5476893		-	-	414988				5891881	
2007	Candidates	Kibaki		Raila	-	Nazlin	Muiru	Matiba	karani, Ng'ethe, Kukubo		(5)
	Votes	4578034		4352860	-	8624	9665	8049	33070		
	Total votes	4578034		4352860	-		59408			9870201	
2013	Candidates	Uhuru		Raila	-	Musalia	Others				(6)
	Votes	6173433		5340546	-	478517	183504				
	Total votes	6173433		5340546	-		662021			12176000	
Totals		20578770		11303328	4408663	2156488				38447249	

Table 2. Routh table

Label			
λ^4	1	b_2	b_4
λ^3	b_1	b_3	0
λ^2	$\frac{-\begin{vmatrix}1 & b_2\\ b_1 & b_3\end{vmatrix}}{b_1} = \frac{b_1b_2-b_3}{b_1}$	$\frac{-\begin{vmatrix}1 & b_4\\ b_1 & 0\end{vmatrix}}{b_1} = b_4$	$\frac{-\begin{vmatrix}1 & 0\\ b_1 & 0\end{vmatrix}}{b_1} = 0$
λ^1	$\frac{-\begin{vmatrix}a_1 & a_3\\ \frac{b_1b_2-b_3}{b_1} & b_4\end{vmatrix}}{\frac{b_1b_2-b_3}{b_1}} = \frac{b_3(b_1b_2-b_3)-{b_1}^2b_4}{b_1b_2-b_3}$	$\frac{-\begin{vmatrix}b_1 & 0\\ \frac{b_1b_2-b_3}{b_1} & 0\end{vmatrix}}{\frac{b_1b_2-b_3}{b_1}} = 0$	$\frac{-\begin{vmatrix}b_1 & 0\\ \frac{b_1b_2-b_3}{b_1} & 0\end{vmatrix}}{\frac{b_1b_2-b_3}{b_1}} = 0$
λ^0	$\frac{-\begin{vmatrix}\frac{b_1b_2-b_3}{b_1} & a_4\\ \frac{b_3(b_1b_2-b_3)-{b_1}^2b_4}{b_1b_2-b_3} & 0\end{vmatrix}}{\frac{b_3(b_1b_2-b_3)-{b_1}^2b_4}{b_1b_2-b_3}} = b_4$	$\frac{-\begin{vmatrix}\frac{b_1b_2-b_3}{b_1} & 0\\ \frac{b_3(b_1b_2-b_3)-{b_1}^2b_4}{b_1b_2-b_3} & 0\end{vmatrix}}{\frac{b_3(b_1b_2-b_3)-{b_1}^2b_4}{b_1b_2-b_3}} = 0$	$\frac{-\begin{vmatrix}\frac{b_1b_2-b_3}{b_1} & 0\\ \frac{b_3(b_1b_2-b_3)-{b_1}^2b_4}{b_1b_2-b_3} & 0\end{vmatrix}}{\frac{b_3(b_1b_2-b_3)-{b_1}^2b_4}{b_1b_2-b_3}} = 0$

Year	Candidate	Political party	Candidate tribe	Support	Total valid votes	Source
1992	Daniel Moi (Incumbent)	KANU	Kalenjin	93% Kalenjin, 79%Mijikenda, 78% Somali, 78% Maasai, 35% Luhya and others		(7)
	Kenneth Matiba	FORD-Asili	Kikuyu	58% Kikuyu, 40% Luhya and others		
	Mwai Kibaki	Democratic Party	Kikuyu	35% Kikuyu, 73% Meru,25% Kisii, 5% Luhya		
	Jaramogi	FORD-Kenya	Luo	95% Luo, 22% Luhya		
	Others					
1997	Daniel Moi (Incumbent)	KANU	Kalenjin	90% Kalenjin, 70% Mijikenda, 73% Somali, 77% Maasai, 40%Luhya and others		(7)
	Mwai Kibaki	Democratic Party	Kikuyu	85% Kikuyu, 48% Kisii, 60% Meru and others		
	Raila Odinga	National Democratic Party	Luo	84% Luo and others		

Year	Candidate	Political party	Candidate tribe	Support	Total valid votes	Source
	Michael Wamalwa	FORD-Kenya	Luhya	52% Luhya		
	Charity Ngilu	Social Democratic Party	Kamba	64% Kamba		
	Others					
2002	Mwai Kibaki	NARC	Kikuyu	68% Kikuyu, 77% Luhya, 93%, Luo, 78% Kamba, 25% Kalenjin, 65% Mijikenda, 73% Somali, 50% Maasai and others		(7)
	Uhuru Kenyatta	KANU	Kikuyu	30% Kikuyu, 67% Kalenjin, 64% Somali, 45% Maasai and others		
	Simeon Nyachae	FORD-People	Kisii	85% Kisii		
	Others					
2007	Mwai Kibaki (Incumbent)	PNU	Kikuyu	93% Kikuyu, 34% Luhya, 50% Somali, 30% Maasai, 15% Kalenjin, 53% Kisii, 30% Mijikenda		
	Raila Odinga	ODM	Luo	99% Luo, 85% Kalenjin, 63% Luhya, 63% Mijikenda		
	ODM-Kenya	ODM-Kenya	Kamba	83% Kamba		
	Others					
2013	Uhuru Kenyatta	Jubilee Coalition	Kikuyu			
	Raila Odinga	CORD	Luo			
	Mudavadi	Amani	Luhya			
	Others					

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