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Quadrature formulas for integral equations of kinetics and digital reactimeters

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Abstract

The aim of this work is to derive quadrature formulas for nuclear reactor kinetic equations in the form of Volterra integral equations of the second kind and reactimeter equations in the form of integral convolution, the kernel of which is a decay function of delayed neutron precursors (DNP) in the non-group form. The expediency of the transition to integral equations is caused by the unification of the direct (calculation of power dynamics) and the reverse (calculation of current reactivity) tasks of reactor kinetics. As a result, the solution is reduced to the calculation of the delayed neutrons integral (DNI). This eliminates the source of computational-experimental discrepancies in estimations of reactivity, which is due to the difference in computational algorithms of direct and inverse problems. The paper describes a general scheme for converting different transport equation approximations to describe the contribution of delayed neutrons by means of an integral convolution without using dynamic equations of the DNP concentration. This conversion reduces the model dimension, simplifies the software implementation, eliminates the stiffness problem of differential kinetic equations and provides the stability of calculations. The model dimension is preserved in the case of several fissile nuclides. The integral form of the equations makes it possible to use the experimental decay function in quadrature formulas, which can be identified in the operating conditions of a nuclear reactor and stored pointwise in a nongroup form without decomposition into the sum of exponentials. This eliminates the need to solve the non-linear problem of identifying group parameters of delayed neutrons and increases the adequacy of modeling. A series of quadrature formulas for the calculation of the DNI are obtained and the corresponding algorithms of a digital reactimeter and numerical simulation of the reactor kinetics are described.

Keywords: Dynamics of nuclear reactor; Point kinetics; Reactivity; Reactimeter; Integral equations; Quadrature formulas.

Introduction

In nuclear reactor physics much attention is paid to a comparison and correlation of calculated and experimental estimations of reactivity [1–4]. Such a comparison characterizes the accuracy and adequacy of neutron-physical calculations in the design, operation, and maintenance of nuclear safety of NPPs. However, as is known [5], differential equations for the description of the dynamics of delayed neutron precursors (DNP) are used in computational complexes, and the experimental estimation of reactivity is based on various versions of the inverse kinetics equation, in which it is easy to show that the contribution of DNPs is described by the convolution integral. Accordingly, various schemes for solving differential equations are used for calculations in the first case [6–8] and the simplest quadrature formulas in the second case [9]. The difference between mathematical models is one of the reasons for the discrepancy between the calculated and experimental results. To eliminate this factor, it is advisable to unify the computational models to ensure the identity of the solution schemes of the direct (power output calculation) and inverse (current reactivity calculation) tasks of the nuclear reactor neutron dynamics. Since the reactivity can be measured only by calculating the integral, the reactivity estimation in the computational modeling complexes must also be performed on the basis of integral equations using quadratures similar to those used in a digital reactimeter. More precisely,

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the equations must be transformed to integral form, and the subsequent discretization should be performed in the same way in both the direct and inverse problems using both traditional quadratures and other known approaches to solving integral equations [10,11].

The paper describes a general scheme for the rearrangement of various approximations of the transport equation to account for the contribution of delayed neutrons by means of the convolution integral. The proposed unification reduces the direct and inverse problems of kinetics to the calculation of the delayed neutron integral (DNI). A series of quadrature formulas for the calculation of the DNI are obtained and the corresponding design schemes for the realization of a digital reactimeter and numerical simulation of the kinetics of the reactor are described. The stability condition for computations is found.

Integro-differential and integral equations of neutron kinetics have long been used in modeling nuclear reactors [6–8,12–26]. The unification of the direct and inverse problems of nuclear reactor kinetics considered in this paper seems to provide a number of improvements in addition to the traditional approaches, namely:

- the model dimension decreases, only the observed quantities appear in the model;
- it becomes possible to use directly, as the kernel of the integral equation, the samples of the experimental DNP decay function;
- the transition to integral equations removes the problem of the stiffness of differential equations of nuclear reactor kinetics;
- it becomes possible to obtain interval estimations of reactivity on the basis of upper and lower integral sums [27];
- the standard metrological analysis schemes [28], based on convolution equations, become applicable for an analysis of reactimeter errors [29].

Unification of the direct and reverse problems of nuclear reactor dynamics

The integral representation of the source of delayed neutrons in the non-stationary transport equation is well known [17,30] and is written on the basis of the concept of the exponential character of decay of delayed neutron precursors in the following form (hereinafter all designations are standard):

$$Q^D(\mathbf{r},\mathbf{v},\tau)=\int_0^t \varphi(\mathbf{r},\mathbf{v},\tau)\sum_{j=1}^{J}\chi_j\beta_j\lambda_j e^{-\lambda_j(t-\tau)}d\tau + \sum_{j=1}^{J}\lambda_j c_j(\mathbf{r},0)\cdot e^{-\lambda_j t}. \quad (1)$$

Integral Summands (1) are solutions of the dynamic equations of DNP concentrations in the corresponding groups:

$$\lambda_j c_j(\mathbf{r},t)=-\frac{\partial c_j(\mathbf{r},t)}{\partial t}+\int \beta_j(\upsilon')\nu_j(\upsilon') \times \Sigma_{fj}(\mathbf{r},\mathbf{v}')\varphi(\mathbf{r},\mathbf{v}',t)d\mathbf{v}', \quad (2)$$

so that, in fact, these equations can be excluded from the computational schemes of the nonstationary transport equation since there is no special interest in the dynamics of DNP concentrations. We shall describe the elimination procedure for the non-stationary transport equation represented in the general form:

$$\frac{1}{\upsilon}\frac{\partial\varphi(\mathbf{r},\mathbf{v},t)}{\partial t}=R\varphi(\mathbf{r},\mathbf{v},t)-\sum_j\chi_j(\upsilon)\frac{\partial c_j(\mathbf{r},t)}{\partial t}. \quad (3)$$

Here, the change in DNP concentrations is taken into account by the second term, and the operator R combines all other processes and is interpreted as a reactivity operator. This equation is obtained by replacing the generation rate of delayed neutrons $\lambda_j c_j$, which appears in the traditional form of the transport equation, by the expression for $\lambda_j c_j$ from Eq. (2).

The initial concentrations of the precursors $c_j(\mathbf{r},0)$ are determined from Eq. (2) under the assumption of the reactor stationary state, i.e., when $\partial c_j/\partial t=0$. Therefore, it is convenient to introduce into Eq. (2) the variable $s_j \equiv \partial c_j/\partial t$, for which these equations take the form of the balance of accelerations of decay (generation) of the precursors:

$$\frac{\partial s_j(\mathbf{r},t)}{\partial t}=-\lambda_j s_j(\mathbf{r},t)+\int\beta_j(\upsilon')\nu_j(\upsilon') \times\Sigma_{fj}(\mathbf{r},\mathbf{v}')\psi(\mathbf{r},\mathbf{v}',t)d\mathbf{v}',$$

where $(\mathbf{r},\mathbf{v},t)\equiv\partial\boldsymbol{\rho},\mathbf{v},t)/\partial t$ is the rate of change of the neutron flux density. The initial condition here becomes zero, $s_j(\mathbf{r},0)\equiv 0$, causing the following kind of solution:

$$s_j(\mathbf{r},t)=\int_0^t e^{-\lambda_j(t-\tau)}\left[\int\beta_j(\upsilon')\nu_j(\upsilon')\Sigma_{fj}(\mathbf{r},\mathbf{v}')\psi(\mathbf{r},\mathbf{v}',\tau)d\mathbf{v}'\right]d\tau.$$

As a result, the problem of calculating the initial distribution of delayed neutron precursors is eliminated and the corresponding source of errors is eliminated.

Substituting $s_j\equiv\partial c_j/\partial t$ in Transfer Eq. (3) brings the latter into the form:

$$\frac{1}{\upsilon}\psi(\mathbf{r},\mathbf{v},t)=R\varphi(\mathbf{r},\mathbf{v},t)-I_{\text{çí}}(\mathbf{r},t)+Q \quad (4)$$

with the initial condition $(\mathbf{r},\mathbf{v},t)=(R(\mathbf{r},\mathbf{v},0)+Q)$. The contribution of delayed neutrons in Transport Eq. (4) is represented by the DNI:

$$I_{3\text{H}}(\mathbf{r},t)=\int_0^t\int W\left(\mathbf{r},\mathbf{v}',t-\tau\right)\psi\left(\mathbf{r},\mathbf{v}',\tau\right)d\mathbf{v}'d\tau, \quad (5)$$

the kernel of which is:

$$W(\mathbf{r},\mathbf{v}',t-\tau)=\sum_j\chi_j(\upsilon)e^{-\lambda_j(t-\tau)}\beta_j(\upsilon')\nu_j(\upsilon')\Sigma_{fj}(\mathbf{r},\mathbf{v}').$$

The described procedure for the change of variables is applicable to the elimination of dynamic equations of DNP concentrations in any approximations of the transport equation. In particular, it brings the system of differential point kinetic

equations to a system of integral equations for the reactor power and power change rate:

$$\begin{bmatrix} v(t) \\ n(t) \end{bmatrix} = \begin{bmatrix} -\int_0^t h(t-\tau)(\cdot)d\tau & r(t) \\ \int_0^t (\cdot)d\tau & 0 \end{bmatrix} \cdot \begin{bmatrix} v(t) \\ n(t) \end{bmatrix} + \begin{bmatrix} Q(t) \\ n(0) \end{bmatrix}. \quad (6)$$

Here the reactivity is given in the Λ-scale: $r = \rho/\Lambda = 1/\Lambda - 1/l$. In this scale, reactivity has the meaning of the relative rate of prompt-neutron-induced processes: $r(t) \equiv v_{\text{pn}}(t)/n(t)$ and can be directly compared with the expressed by the reverse period $(t) = v(t)/n(t)$ relative reactor power change rate with respect to all the processes. This, to a certain extent, simplifies the solution of the known problem of organizing "period or reactivity" control of the nuclear power unit [31].

The procedure for deriving the integral equation for the power change rate is similar to the transformations performed by the traditional derivation of the reactimeter equation (reversed solution of the kinetics equations). Here the reactimeter equation is obtained from the first Eq. (6) obviously without usually involved Laplace transforms [17,32]:

$$r(t) = \alpha(t) + \frac{1}{n(t)} \int_0^t h(t-\tau)dn(\tau) + Q(t)/n(t). \quad (7)$$

After integrating by parts, Eq. (7) takes on the form:

$$r(t) = \alpha(t) + h(0) - \frac{1}{n(t)} \left[h(t)n(0) + \int_0^t n(t-\tau)dh(\tau) + Q(t) \right], \quad (8)$$

where $h(0) = \beta_{\text{eff}}/\Lambda$; $g(\tau) = dh(\tau)/d\tau$. For operational modes, when, as a rule, $h(0) >> (t) + Q(t)/n(t)$ and the observation interval $[0,t]$ exceeds the function decay time $h(t)$, Eq. (8) takes on the form:

$$r(t) = \frac{\beta_{\text{эфф}}}{\Lambda} - \frac{1}{n(t)} \int_0^t g(t-\tau)n(\tau)d\tau. \quad (9)$$

In all cases, the procedures for solving both the direct and inverse problems of NR kinetics are unified since they are reduced to the calculation of the delayed neutron integral. In this case, Eqs. (8) and (9) are more preferable for numerical realization since the decay time of the function $g(\tau) = dh(\tau)/d\tau$, appearing in these equations, is approximately three times less than the decay time of the function $h(t)$. On the other hand, it is more convenient to identify the function $h(t)$ on the basis of Eq. (7).

Discreteization of delayed neutron integral

The kernel of the DNI in Eqs. (5) and (6) is (up to a factor $1/\Lambda$ the decay function of delayed neutron precursors usually represented in exponential notation:

$$h(t) = \frac{\beta_{\text{eff}}}{\Lambda} \sum_{j=1}^{J} \alpha_j \exp(-\lambda_j t).$$

Instead of exponential notation of the kernel, any approximation or pointwise tabular storage of the experimental decay curve is permissible [33]. Such an approach increases the model adequacy, simplifies the procedures for adapting the reactimeter, but requires the calculation of the DNI according to schemes in which the kernel is represented by a finite set of samples of the experimental decay curve or its derivative.

Let us consider the discrete DNI form as applied to Eqs. (8) and (9), assuming that the function $g(t)$ is represented by the $L+1$ sample (i.e., $g(t) = 0$ for $t > t_L$):

$$I_k = \sum_{l=0}^{L} c_{k,k-l} g_{k-l} n_l = \sum_{l=k-L}^{L} c_{k,k-l} g_{k-l} n_l = \sum_{l=0}^{L} c_{k,l} g_l n_{k-l}. \quad (10)$$

The calculation of the convolution integral in Form (10) is a classical problem of digital signal processing considered, in particular, in [34–36]. However, the corresponding approaches were practically not used for calculating the DNI in the digital reactimeter equation or solving the direct problem of NR kinetics.

In the case of the conventionally used trapezoid formula, the quadrature coefficients in (10) are $c_{k,0} = T_{k-1}/2$; $c_{k,l} = (T_{k-l} + T_{k-l-1})/2$; $l = 1, \ldots, L-1$; $c_{k,L} = T_{k-L}/2$, if the integration step $T_l = t_{l+1} - t_l$ is variable. It can be seen that the estimation from the trapezoid formula is obtained by averaging the estimations made according to the formulas of the left and right rectangles, which can be used for an interval estimation of the solution, for example, in reactivity calculations [27].

Similar to the trapeziod formula, Simpson's quadrature formula describes simple averaging of three estimations obtained by the method of left, right, and central rectangles:

$$S_1 = T_{l2}(f_l + f_{l+1})/2, \quad S_2 = T_{l2} f_{l+1},$$
$$S_3 = T_{l2}(f_{l+1} + f_{l+2})/2, \quad T_{l2} = t_{l+2} - t_l,$$

where S_2 is the integral estimation on the interval $[t_l, t_{l+2}]$ by the average rectangle method, and S_1, S_3 are the pairwise averaging of the estimation S_2 with the integral estimations by the left and right rectangle methods. As for the calculation of Convolution (10) in the case of a variable integration step, Simpson's formula on the lth interpolation interval ($l = 0,2,4,\ldots, L-2$) has the form:

$$S_l = \frac{T_{i-2} + T_{i-1}}{6} \left(\frac{2T_{i-2} - T_{i-1}}{T_{i-2}} f_{i-2} + \frac{(T_{i-2} + T_{i-1})^2}{T_{i-2} T_{i-1}} f_{i-1} + \frac{2T_{i-1} - T_{i-2}}{T_{i-1}} f_i \right),$$
$$i = k - l, \; f_i = g_{k-i} n_i.$$

To apply Simpson's quadrature, the number of samples $L+1$ must be odd. To eliminate this difficulty, we shall use the above interpretation of quadratures as averaging formulas for elementary intervals. In the general case, this leads to the formula [37]

$$S_i^l = \left(\mathbf{q}_i^l\right)^T \mathbf{W}_l^{-1} \mathbf{f}_l, \quad (11)$$

that evaluates the integral on the ith elementary interval $[t_i, t_{i+1}]$, $i = 0, \ldots, k-1$ from information related to the interpolation interval $T_{l,l+J} = [t_l, t_{l+1}, \ldots, t_{l+J}]$, $l = \max(0, i+1-J)$,

…, min(*i*,*k*–*J*), enclosing a given elementary interval, i.e., by the distribution of the nodes $[t_l,t_{l+1},\ldots,t_{l+J}]$ and the integrand value vector $\mathbf{f}_l=(f(t_{l+m}))^T$, $m=0, \ldots, J$. The matrix $\mathbf{W}_l$ and the vector $\mathbf{q}^l{}_i$ are computed in the local coordinates $x=t-t_l$ of the *l*th interpolation interval. The matrix entries $w^l{}_{jm}=w_j(x_i)$, $x_i=t_i-t_l$ are the values of the basis functions of the interpolation formula:

$$f(x)=\sum_{j=0}^{n} a_j^l w_j(x),$$

calculated in the indicated nodes, and the elements of the vector $\mathbf{q}^l{}_i$ are the integrals of the basis functions over the *i*th elementary interval:

$$q_j^l=\int_{x_i}^{x_{i+1}} w_j(x)dx.$$

Specific quadrature formulas are obtained from (11) by averaging over this or that combination of $L(i)$ interpolation intervals covering the *i*th elementary interval:

$$S_i\approx\left(\sum_{l=\max(0,i+1-J)}^{\min(i,k-J)}(\mathbf{q}^l{}_i)^T\mathbf{W}_l{}^{-1}f_l\right)/L(i).$$

For Simpson's quadrature formula on the interval $X_{l,l+2}=[x_l, x_{l+1}, x_{l+2}]$, the matrix

$$\mathbf{W}_l^{-1}=\begin{bmatrix}1 & 0 & 0\\ -\left(\dfrac{1}{t_{l,l+2}}+\dfrac{1}{t_l}\right) & \left(\dfrac{1}{t_{l+1}}+\dfrac{1}{t_l}\right) & \left(\dfrac{1}{t_{l,l+2}}-\dfrac{1}{t_{l+1}}\right)\\ \dfrac{1}{t_l t_{l,l+2}} & -\dfrac{1}{t_l t_{l+1}} & \dfrac{1}{t_{l+1}t_{l,l+2}}\end{bmatrix},\tag{12}$$

where indexation is used in the following sense: $x_{=}t_{+1}-t$; $x_{,=}t_{\square}t$. In this case, the *i*th elementary interval (except for the initial and final ones) belongs only to two interpolation intervals: (*i*–1)th and *i*th, so that the average estimation of the integral on the *i*th elementary interval is:

$$S_i=\left(\left(\mathbf{q}_i^{i-1}\right)^T\mathbf{W}_{i-1}^{-1}\mathbf{f}_{i-1}+\left(\mathbf{q}_i^i\right)^T\mathbf{W}_i^{-1}\mathbf{f}_i\right)/2;\quad i=1,\ldots,m-2,\tag{13}$$

where the integral vectors of the basis functions are equal to:

$$\mathbf{q}_i^i=x_i\left[1\quad (x_i/2)\quad \left(x_i^2/3\right)\right]^T,$$
$$\mathbf{q}_i^{i-1}=x_{i-1,i+1}\left[1\quad \left(x_{i-1,i+1}/2\right)\quad \left(x_{i-1,i+1}^2/3\right)\right]^T - x_{i-1,i}\left[1\quad \left(x_{i-1,i}/2\right)\quad \left(x_{i-1,i}^2/3\right)\right]^T,$$

and indexation is similar to that adopted in formula (12).

With a constant integration step $x_i=T$, the matrix of values of the basis functions at the nodes is the same for all interpolation intervals:

$$\mathbf{W}^{-1}=\mathrm{diag}\left[1;\quad 1/T;\quad 1/T^2\right]\cdot\begin{bmatrix}1 & 0 & 0\\ -3/2 & 2 & -1/2\\ 1/2 & -1 & 1/2\end{bmatrix},$$

and the integral vectors of the basis functions over the elementary intervals are $\mathbf{q}^i{}_i=T[1\ (T/2)\ (T^2/3)]^T$ and $\mathbf{q}^{i-1}{}_i=T[1\ (3T/2)\ (7T^2/3)]^T$, respectively. As a result, Estimation (13) takes the form: $S_i=T(-f_{i-1}+13f_i+13f_{i+1}-f_{i+2})/24$. Unlike the standard Simpson's integration scheme, the application of this formula does not require a certain multiplicity of the number of nodes. If there are no grounds for choosing concrete values of the integrand outside the complete integration interval $[t_{k-L},t_k]$, then, on the finite elementary interval $[t_{k-1},t_k]$, we should apply an estimation over the left interpolation interval $S_{k-1}=T(-f_{k-2}+8f_{k-1}+5f_k)/12$ and, on the initial interval $[t_{k-L},t_{k-L+1}]$, over the right interpolation interval $S_0=T(5f_0+8f_1-f_2)$. In this case, applying Estimation (13) on the remaining elementary intervals $[t_i,t_{i+1}]$; $i=k-L+1, \ldots, k-2$, we obtain the complete quadrature formula for the interval $[t_{k-L},t_k]$:

$$S_{k-L,k}=\frac{T}{24}\left[9f_{k-L}+28f_{k-L+1}+23f_{k-L+2}+23f_{k-2}+28f_{k-1}+9f_k\right]+T\sum_{i=k-L+3}^{k-3}f_i.$$

Let us give concrete variants of digital realization of Reactimeter Eq. (7) determined by the choice of quadrature formulas.

(1) The reactimeter equation in calculating the DNI by the method of trapezoids:

$$r_k=\alpha_k\left(1+\frac{T}{2}h_0\right)+\frac{T}{n_k}\sum_{l=1}^{k-1}h_l v_{k-l}-\frac{Q}{n_k}.$$

(2) The reactimeter equation with moving integration according to Simpson's formula:

$$r_k=\alpha_k\left(1+\frac{5T}{12}h_0\right)+\frac{T}{12n_k}\left(13h_1v_{k-1}+12\sum_{l=2}^{k-3}h_l v_{k-l}+11h_{k-2}v_2+15h_{k-1}v_1+4h_k v_0\right)-\frac{Q}{n_k},\ k=4,5,6,\ldots$$

or for a fixed number of samples of the DNP decay function:

$$r_k=\left(1+\frac{5T}{12}h_0\right)\alpha_k+\left(\frac{13}{12}\cdot\frac{T}{n_k}\right)h_1v_{k-1}+\frac{T}{n_k}\sum_{l=2}^{L}h_l v_{k-l}-\frac{Q}{n_k}.$$

(3) The reactimeter equation in the case of integration by Newton's method:

$$r_k=\alpha_k+\frac{3T}{8n_k}\left(h_{k-1}v_1+\frac{28}{9}h_{k-2}v_2+\frac{23}{9}h_{k-3}v_3+\sum_{l=3}^{k-4}h_l v_{k-l}+\frac{23}{9}h_2v_{k-2}+\frac{28}{9}h_1v_{k-1}+h_0v_k\right)-\frac{Q}{n_k},\ k=1,2,\ldots$$

For a fixed number of the decay function samples, the equation takes on the form:

$$r_k = \left(1 + \frac{3T}{8}h_0\right)\alpha_k + \left(\frac{7}{6}\cdot\frac{T}{n_k}\right)h_1 v_{k-1} + \left(\frac{23}{24}\cdot\frac{T}{n_k}\right)h_2 v_{k-2} + \frac{3T}{8n_k}\sum_{l=3}^{L} h_l v_{k-l} - \frac{Q}{n_k}.$$

Quadrature formulas for integral kinetic equations

Discrete analogues of Eq. (6) have the form:

$$v_k = -\sum_{l=0}^{k}(a_{k,l}h_{k-l})v_l + r_k n_k + Q_k,\ n_k = n_0 + \sum_{l=0}^{k} b_{k,l}\cdot v_l, \tag{14}$$

providing the following general computational scheme:

$$w_1 \equiv n_0,\ w_k = n_0 + \sum_{i=0}^{k-1} b_{k,l}v_l,$$
$$v_k = d_k\left(Q_k - \sum_{l=1}^{k-1} a_{k,l}h_{k,l}v_l + r_k w_k\right),\ n_k = w_k + b_{k,k}v_k,\ k = 1, 2, 3, \ldots \tag{15}$$

where the coefficient $d_k = (1 + a_{k,k}h_0 - r_k b_{k,k})^{-1}$. Henceforward, it is assumed that the calculation is carried out for the case of a start from the reactor stationary state, so $v_0 = r_0 n_0 + Q_0$. This initial condition makes it possible to describe, in the framework of one computational scheme, any scenario of a change in power or reactivity [12,33]. In applied problems, both the reactor power and period $p = n/v$ are of interest, so calculation by Algorithm (15) seems to be the most natural. Further detailing is determined by the selection of specific quadratures.

For suppressing the accumulation of errors, the coefficient d_k should be less than unity. This is always the case in a subcritical reactor. In a supercritical reactor, the condition for the value of the reactivity introduced should be satisfied: $r_k < (a_{k,k}/b_{k,k})\cdot h_0$. This relationship indicates that it is advisable to use its own type of quadrature formula for each of Eq. (14). Thus, if $b_{k,k} = 0$, then $r_k < \infty$, i.e., the restriction on the reactivity introduced is not available when the open quadrature formula is used to calculate the power. This scheme takes place when, for example, the left-hand rectangle formula is used to calculate the power and the right-hand rectangle function is used to calculate the rate:

$$d_k = (1 + T_{k-1}h_0)^{-1},\quad nk = n_{k-1} + T_{k-1}v_{k-1},$$
$$v_k = d_k\left(Q_k + r_k n_k - \sum_{l=1}^{k-1} T_{l-1}h_{k-l}v_l\right),\quad k = 1, 2, \ldots$$

Applying Simpson's formula and averaging over two interpolation intervals, which in this case may belong to an elementary interval, we obtain the following quadrature formula to calculate the power:

$$n_k = n_0 + \frac{T}{12}\left(4v_0 + 15v_1 + 11v_2 + 12\sum_{l=3}^{k-2} v_l + 13v_{k-1} + 5v_k\right),\quad k \geq 4.$$

If the delayed neutron integral is calculated using the analogous formula, General Algorithm (15) is concretized as follows:

$$w_k = n_0 + \frac{T}{12}\left(15v_1 + 11v_2 + 12\sum_{l=3}^{k-2} v_l + 13v_{k-1}\right),$$
$$d_k = (1 + 5T(h_0 - r_k)/12)^{-1},$$
$$v_k = d_k\left(Q_k - \frac{T}{12}\left(15h_{k,1}v_1 + 11h_{k,2}v_2 + 12\sum_{l=3}^{k-2} h_{k,l}v_l + 13h_{k,k-1}v_{k-1}\right) + r_k w_k\right),$$
$$n_k = w_k + \frac{5T}{12}v_k,\ k = 4, 5, 6, \ldots$$

with the error-suppressing condition: $r_k < h_0$.

If the DNI is calculated by the trapezoidal formula, we obtain the following computational scheme:

$$w_k = n_0 + \frac{T}{12}\left(15v_1 + 11v_2 + 12\sum_{l=3}^{k-2} v_l + 13v_{k-1}\right),$$
$$v_k = \frac{Q_k - \frac{T}{2}\sum_{l=1}^{k-1} h_{k,l}v_l + r_k w_k}{(1 + T(6h_0 - 5r_k)/12)},\ n_k = w_k + \frac{5T}{12}v_k,\ k = 4, 5, 6, \ldots$$

with an error-suppressing condition $r_k < 1.2h_0$ weaker than in the previous scheme. This again confirms the reasonability of using non-coincident types of quadrature formulas in Eq. (6).

Conclusions

(1) A model of nuclear reactor kinetics is proposed in the form of a system of Volterra integral equations of the second kind for the power and power change rate, which unifies the direct and inverse problems of kinetics by reducing them to the calculation of the delayed neutron integral.
(2) A series of quadrature formulas for the DNI is obtained in the case of non-group representation of the DNP decay function and the corresponding digital reactimeter equations are presented.
(3) A description is given of the general computational scheme for the proposed model. The algorithm is concretized for the cases of using a number of popular quadrature formulas.
(4) For step-by-step application of quadrature formulas requiring a certain multiplicity of the number of interpolation nodes, a moving integration algorithm is proposed. The corresponding quadrature formulas are obtained with respect to the DNI calculation.

(5) Conditions for suppressing errors in integration are indicated.

The implementation of the proposed algorithms can be exemplified by the results presented in [27,29,33]. However, all possible combinations of quadrature types used to discretize the model equations as well as different averaging options in moving integration schemes lead to numerous variants of numerical implementation, which requires a further analysis and comparison by accuracy and number of operations. It is necessary to perform such a comparison with other known algorithms for modeling nuclear reactor kinetics.

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