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Improving Bandwidth of Yagi-Uda Arrays

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ABSTRACT

A novel approach for improving antenna bandwidth is described using a 6-element Yagi-Uda array as an example. The new approach applies Central Force Optimization, a deterministic metaheuristic, and Variable Z_0 technology, a novel, proprietary design and optimization methodology, to produce an array with 33.09% fractional impedance bandwidth. This array's performance is compared to its CFO-optimized Fixed Z_0 counterpart, and to the performance of a 6-element Dominating Cone Line Search-optimized array. Both CFO-optimized antennas exhibit better performance than the DCLS array, especially with respect to impedance bandwidth. Although the Yagi-Uda antenna was chosen to illustrate this new approach to antenna design and optimization, the methodology is entirely general and can be applied to any antenna against any set of performance objectives.

Keywords: Variable Z_0; Z_0; Characteristic Impedance; Feed System; Antenna; Antenna Design; Antenna Optimization; Design Objectives; Performance Objectives; Bandwidth; Impedance Bandwidth; Broadband; Ultra Wideband; UWB; Yagi; Yagi-Uda; Central Force Optimization; CFO; Numerical Optimization; Optimization Algorithm; Metaheuristic; Dominating Cone Line Search; DCLS

1. Introduction

This note describes a novel approach to improving antenna bandwidth using a six-element Yagi-Uda array as an example. Developed more than 85 years ago [1,2], the "Yagi" still is widely used, but it is inherently narrowband, to quote: "Usually Yagi-Uda arrays have low input impedance and relatively narrow bandwidth (on the order of about 2%)" [3, p.396]. Modern well-designed Yagis achieve greater bandwidth, on the order of 5% [4] to more than 15% [5], but these bandwidths still are far below the requirements of many wireless systems. The Federal Communications Commission, for example, defines an Ultra Wideband (UWB) antenna as having a fractional impedance bandwidth (IBW) of at least 20%, or an absolute bandwidth of 500 MHz [6, p.15]. Central Force Optimization (CFO) and Variable $Z_0^{(\mathrm{sm})}$ [1] technology (Var Z_0, VZ_0) are applied to the Yagi design problem, and the array's IBW performance is compared to another state-of-the-art design. The CFO-VZ_0 Yagi achieves a very robust IBW of 33.09% for VSWR ≤ 2:1.

2. Yagi-Uda Array

The 6-element array comprises a center-fed driven element (DE) excited by a radio-frequency (RF) source flanked by a parasitic reflector (REF) on one side and four parasitic directors (D_n, $n = 1, \cdots, 4$) on the other. All elements are PEC (Perfectly Electrically Conducting). Three Yagis are described in this paper. Two were optimized with CFO, one using Variable Z_0, which treats Z_0 as an unknown variable whose value is to be determined by the optimization algorithm [7,8], and the other using the traditional approach of assigning Z_0 a fixed value (50 Ω resistive in this case). These two antennas are referred to, respectively, as CFO-VZ_0 and CFO-FZ_0. CFO was selected as the optimization algorithm because it has performed well against recognized benchmark functions and problems in applied electromagnetics [9-18] and is a useful tool for antenna optimization.

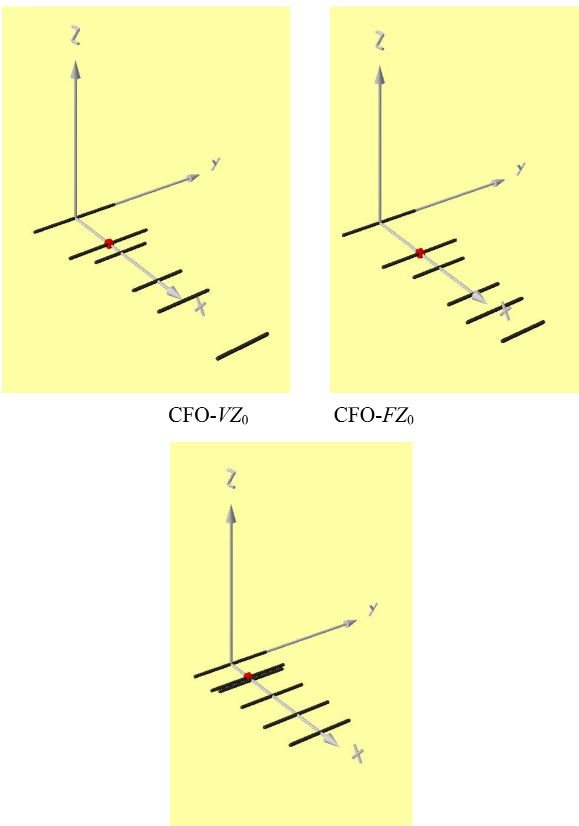
The third antenna was optimized with Dominating Cone Line Search (DCLS) [5] and therefore provides a state-of-the-art comparison. The "A3" Yagi, whose dimensions are adapted from in table IV in [5] (REF moved to the Y-axis), was chosen for comparison because it has the largest bandwidth of the three antennas discussed in that paper. Both CFO and DCLS are deterministic algorithms, thus providing the major advantage compared to stochastic algorithms of returning the same results for every run with the same setup.

The Yagis' performance was computed using NEC-4.2D (Numerical Electromagnetics Code ver. 4.2 double

[1]Variable Z_0, V Z_0, and Var Z_0 are trademarks and service marks of Massachusetts Corporation Variable Z_0, Ltd., P.O. Box 1714, Harwich, MA 02645 USA.

precision) [19-21]. NEC is a widely used Method of Moments (MoM) wire structure modeling program developed at Lawrence Livermore National Laboratory (LLNL). A freeware version of NEC-2 is available online (source code, executables and a GUI) [22,23]. The modeled antennas are shown in **Figure 1** (visualized using 4nec2 [22], the red circle indicating the RF source with the axis length being 1 meter for scaling). Each design's geometry is quite different.

Table 1 shows element lengths and positions along the boom (+X-axis). Dimensions are in wavelengths, λ_0, at the design frequency, f_0. The corresponding NEC input files appear in **Figure 2**. Because NEC requires dimensions in meters, f_0 was chosen to be 299.8 MHz corresponding to a wavelength of $\lambda_0 = 1$ meter. The Yagi designs therefore can be scaled to any frequency because dimensions in meters are in wavelengths at f_0. The element radius was set to 0.009097 meter (or λ_0) because that value is used in [5]. Note that the design frequency f_0 and the band center frequency, f_C, discussed below are not (necessarily) the same. Even though the array is designed at a particular frequency, the optimized design's best performance may be (likely is) in a band whose center frequency is different.



CFO-VZ_0 CFO-FZ_0 A3

Figure 1. 6-element yagis.

Table 1. Yagi element lengths & boom positions.

Array Element	CFO-VZ_0		CFO-FZ_0		A3	
	$L(\lambda_0)$	$X(\lambda_0)$	$L(\lambda_0)$	$X(\lambda_0)$	$L(\lambda_0)$	$X(\lambda_0)$
REF	0.578	0	0.516	0	0.5072	0
DE	0.540	0.348	0.508	0.410	0.5070	0.1808
D_1	0.346	0.461	0.340	0.585	0.4416	0.2144
D_2	0.324	0.789	0.334	0.885	0.4169	0.4159
D_3	0.332	1.006	0.364	1.056	0.4325	0.6397
D_4	0.326	1.443	0.270	1.267	0.3952	0.8405

3. CFO-Variable Z_0

The objective function (fitness) to be maximized by CFO was defined as

$$F(\vec{x}) = \sum_{k=1}^{3} c_k \cdot G_{fwd}(f_k, \vec{x}) - \sum_{i=1}^{3} c_i \cdot \text{VSWR} \,//\, Z_0(f_i, \vec{x})$$

where f is the frequency and $\vec{x}$ the decision vector defined as

$$\vec{x} = \big(Z_0, L_{REF}, L_{DE}, L_{D_1}, L_{D_2}, L_{D_3}, L_{D_4}, X_{DE}, X_{D_1}, X_{D_2}, X_{D_3}, X_{D_4}\big)$$

The Yagi optimization problem is 12-dimensional with the following decision variables: Z_0, the feed system characteristic impedance (or source internal impedance if there is no feed line); and the eleven geometric variables corresponding to the Yagi element lengths ("L") and boom coordinates ("X"), each subscripted with the corresponding element name (note that REF is placed symmetrically on the *Y*-axis, that is, at $X = 0$). The fitness increases with increasing forward gain G_{fwd} and decreasing VSWR $// Z_0$, VSWR $// Z_0$ being the voltage standing wave ratio relative to Z_0 (// denotes "relative to"). The fitness is evaluated at three frequencies as shown in **Table 2** using the empirically determined coefficients c_i and c_k. G_{fwd} is evaluated in the direction of the + X-axis ($\theta = 90^\circ$, $\varphi = 0^\circ$ in NEC's right-handed spherical polar coordinate system). Algorithm details and a complete source code listing are available online [8], and an electronic listing is available upon request to the author (rf2@ieee.org).

The objective function is designed to maximize gain and bandwidth. The term "bandwidth" refers generally to the range of frequencies over which some specific an-

Table 2. Fitness coefficients.

i	c_i	k	c_k	$f_{i,k}$ (*MHz*)
1	5.0	1	0.2	239.8
2	8.0	2	0.8	299.8
3	0.9	3	1.0	359.8


```
CM File: YAGI.NEC
CM CFO-OPTIMIZED YAGI ARRAY IN FREE SPACE
CM Design frequency, Fo = 299.8 MHz
CM Freq step = 60 MHz +/- Fo
CM Run ID: 10022011_092307
CM Fitness function:
CM .2*Gfwd(L)-5*VSWR(L)+.8*Gfwd(M)-8*VSWR(M)+
CM 1*Gfwd(U)-.9*VSWR(U)
CM where L,M,U are lower/mid/upper frequencies
CM Variable Z0=107.91 ohms
CM Note: All dimensions are in METERS.
CM File ID 10022011140515
CM Nd= 12, p= 48, j= 150
CE
GW1,9,0.,-.289,0.,0.,.289,0.,.009097
GW2,9,.348,-.27,0.,.348,.27,0.,.009097
GW3,9,.461,-.173,0.,.461,.173,0.,.009097
GW4,9,.789,-.162,0.,.789,.162,0.,.009097
GW5,9,1.006,-.166,0.,1.006,.166,0.,.009097
GW6,9,1.443,-.163,0.,1.443,.163,0.,.009097
GE
FR 0,2001,0,0,200.,.1
EX 0,2,5,1,1.,0.
RP 0,19,19,1001,0.,0.,5.,10.,100000.
EN
```

(a)

```
CM File: YAGI.NEC
CM CFO-OPTIMIZED YAGI ARRAY IN FREE SPACE
CM Design frequency, Fo = 299.8 MHz
CM Freq step = 60 MHz +/- Fo
CM Run ID: 10022011_141510
CM Fitness function:
CM .2*Gfwd(L)-5*VSWR(L)+.8*Gfwd(M)-8*VSWR(M)+
CM 1*Gfwd(U)-.9*VSWR(U)
CM where L,M,U are lower/mid/upper frequencies
CM Fixed Z0=50 ohms
CM Note: All dimensions are in METERS.
CM File ID 10022011163845
CM Nd= 12, p= 24, j= 141
CE
GW1,9,0.,-.258,0.,0.,.258,0.,.009097
GW2,9,.41,-.254,0.,.41,.254,0.,.009097
GW3,9,.585,-.17,0.,.585,.17,0.,.009097
GW4,9,.885,-.167,0.,.885,.167,0.,.009097
GW5,9,1.056,-.182,0.,1.056,.182,0.,.009097
GW6,9,1.267,-.135,0.,1.267,.135,0.,.009097
GE
FR 0,2001,0,0,200.,.1
EX 0,2,5,1,1.,0.
RP 0,19,19,1001,0.,0.,5.,10.,100000.
EN
```

(b)

```
CM File: Lisboa_YAGI_A3.NEC
CM BROADBAND YAGI ARRAY, Lisboa DESIGN #A3
CM Run ID: 09282011_0915
CM Design frequency, Fo = 299.8 MHz.
CM Z0=50 ohms.
CM Note: All lengths in METERS.
CM File ID 092820110915
CE
GW1,9,0.,-.2536,0.,0.,.2536,0.,.009097
GW2,9,.1808,-.2535,0.,.1808,.2535,0.,.009097
GW3,9,.2144,-.2208,0.,.2144,.2208,0.,.009097
GW4,9,.4159,-.20845,0.,.4159,.20845,0.,.009097
GW5,9,.6397,-.21625,0.,.6397,.21625,0.,.009097
GW6,9,.8405,-.1976,0.,.8405,.1976,0.,.009097
GE
FR 0,2001,0,0,200.,.1
EX 0,2,5,1,1.,0.
RP 0,19,19,1001,0.,0.,5.,10.,100000.
EN
```

(c)

Figure 2. (a) CFO-VZ_0; (b) CFO-FZ_0; (c) A3.

tenna performance measure is met. For example, "gain bandwidth" is the frequency range over which a minimum power gain is achieved, and so on with respect to other performance measures. IBW is defined as the frequency band or bands within which the antenna input impedance, $Z_{in} = R_{in} + jX_{in}$, $j = \sqrt{-1}$, is matched to the feed system characteristic impedance, Z_0, within specified limits. The desired degree of matching can be specified in many ways, for example, in terms of the antenna's actual input impedance (resistance, R_{in}, and reactance, X_{in}) as a function of frequency, or, as is more often the case in practice, in terms of a maximum VSWR. IBW typically is specified as VSWR // Z_0 $\leq$ 2:1, which is equivalent to a return loss (scattering parameter S_{11}) approximately less than –10 dB. Other VSWR thresholds can be used instead, and frequently are. Military systems, for example, often use a 3:1 threshold. Zehforoosh *et al.* [24] describe the design of an UWB microstrip antenna and provide a good summary of Z_0's significance as a design parameter in the context of IBW.

Of course, the antenna designer is free to specify any desired fitness function, and its specific form will produce different antenna designs as a consequence of the different decision space landscape. Some objective functions may introduce Z_0 indirectly as a variable quantity, as is done here through the VSWR; or it may be introduced explicitly as, for example, in the bowtie fitness function in [7], where it is variable, or in the second and third objective functions used in [25], where it is fixed.

The objective of maximizing IBW is more easily met using Variable Z_0 instead of traditional methodology. Var Z_0 is a new, proprietary (patent pending) paradigm for antenna design and optimization methodology that apparently has been heretofore overlooked. For purposes of discussion the term "design" refers to the process of specifying a complete set of parameters defining an antenna meeting specific performance objectives, while "optimization" refers to specifying a complete set of parameters defining the antenna that best meets specific performance objectives. Methodology refers to the methods, techniques, processes, or procedures traditionally used for antenna design and optimization that treat Z_0 as a fixed parameter with a constant value that is assigned at the start of the methodology (even if multiple parametric runs are made). Traditional methodology does not consider Z_0 a variable quantity whose value is determined by the methodology. This distinction is fundamental and quite important. Traditional methodology excludes from the outset all designs that could provide better performance by using some other value of Z_0. The A3 array, for example, was designed against $Z_0 = 50\ \Omega$ resistive, but its performance likely would be better with another value. By treating Z_0 as another design variable in the set of variable antenna system parameters to be determined by the methodology, Var Z_0 improves on traditional methodology by adding another degree of freedom to the design space or, in the case of optimization, the decision space, thereby making it easier to achieve any particular performance objectives. While Var Z_0 is especially useful for increasing IBW, it can be used to achieve any performance objectives, including ones not involving IBW.

Besides CFO, any number of other commonly employed algorithms, such as Particle Swarm (PSO) [26], Ant Colony (ACO) [27], Group Search Optimizer (GSO) [28], Differential Evolution (DE) [29-31], or Genetic Algorithm (GA) [32] could be used instead. This "product by process" approach applies to any methodology, deterministic ones like CFO; stochastic metaheuristics like PSO, ACO, GSO, DE or GA; analytic approaches such as extended Wu-King impedance loading [14]; or even "seat of the pants" design or optimization based on experience, intuition, or a "best guess." The specific design or optimization methodology is irrelevant to the novelty and utility of treating Z_0 as a design variable instead of a fixed parameter. Var Z_0 can be used advantageously with any design or optimization methodology.

4. Results

Figure 3 plots VSWR for the three Yagis, and **Table 3** summarizes the bandwidth data for three different VSWR thresholds (2:1, 2.5:1, and 3:1). In the table, f_L and f_U are the lower and upper frequency limits (MHz) corresponding to VSWRs not exceeding the specified threshold, and $\Delta f = f_U - f_L$ is the bandwidth in MHz. The fractional bandwidth in percent relative to the band center frequency f_C is computed as $\mathrm{BW}(\%) = \dfrac{200\Delta f}{f_L + f_U}$. The CFO-Var Z_0 Yagi exhibits UWB performance (fractional bandwidth ≥ 20%) at all VSWR thresholds, whereas the fixed Z_0 antenna is UWB at 2.5:1 and above. The Var Z_0 approach increased the Yagi's $VSWR \le 2{:}1$ IBW from 19.22% at $Z_0 = 50\ \Omega$ to 33.09% at $Z_0 = 107.91\ \Omega$. This improvement is dramatic, and directly attributable to Var Z_0 because CFO is a deterministic metaheuristic (the improvement thus cannot be a consequence of an optimizer's stochasticity). With respect to matching this array to a "standard" 50 Ω feed system impedance, a 2.16:1 impedance ratio broadband transformer or other suitable matching network is required, which easily is accomplished with state-of-the-art matching techniques. By injecting an additional degree of freedom into the traditional antenna design or optimization methodology, Var Z_0 technol-

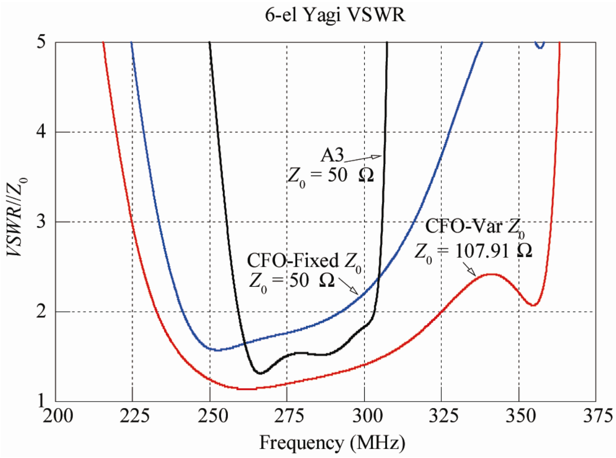


Figure 3. Yagi VSWR relative to Z_0.

Table 3. 6-el Yagi bandwidth.

Yagi Design	f_L	f_C	f_U	Δf	BW (%)
VSWR Threshold 2:1					
CFO-VZ_0 (1)	232.80	278.95	325.10	92.30	33.09
CFO-FZ_0 (2)	241.30	266.95	292.60	51.30	19.22
A3 (3)	259.25	281.23	303.20	43.95	15.63
VSWR Threshold 2.5:1					
Yagi Design	f_L	f_C	f_U	Δf	BW (%)
CFO-VZ_0	228.15	293.63	359.10	130.95	44.60
CFO-FZ_0	236.80	272.15	307.50	70.70	25.98
A3	257.10	281.08	305.05	47.95	17.06
VSWR Threshold 3:1					
Yagi Design	f_L	f_C	f_U	Δf	BW (%)
CFO-VZ_0	224.80	292.70	360.60	135.80	46.40
CFO-FZ_0	233.55	274.83	316.10	82.55	30.04
A3	255.40	280.62	305.83	50.43	17.97

(1) CFO-optimized array, Variable Z_0, $Z_0 = 107.91\ \Omega$. (2) CFO-optimized array, Fixed Z_0, $Z_0 = 50\ \Omega$. (3) DCLS-optimized array (Lisboa A3), Fixed Z_0, $Z_0 = 50\ \Omega$.

ogy has produced a substantially better antenna than the CFO-optimized traditional fixed Z_0 design.

Compared to the CFO-optimized Yagis, the A3 array is considerably more narrowband. Its VSWR ≤ 2:1 IBW is only 15.63%, and it increases only slightly to 17.97% at 3:1. The A3 array's best IBW performance across all three VSWR thresholds is not as good as the CFO-FZ_0 array at its lowest threshold. This is a consequence of the very steep skirts in its VSWR plot. While the CFO-Var Z_0 array exhibits similarly steep skirts, as is evident from the plot, its basic IBW is much greater to start.

Figures 4 and **5** plot forward gain (dBi, decibels relative to isotropic) and front-to-back ratio (FBR), respectively [note that in this case directivity power gain are equal because the PEC array elements result in 100% radiation efficiency]. The CFO-optimized arrays have generally flatter gain curves with moderate gain values compared to the A3 design. While the A3's gain is higher mid-band, the gain bandwidth is narrow, and the gain falls off precipitously with increasing frequency. Both CFO-optimized arrays also exhibit a substantial decrease in gain at the high end of the band, but the drop off occurs well above the highest frequency with acceptable VSWR. The A3 array has a very high mid-band FBR, but only over a fairly narrow range of frequencies. On either side, its FBR decreases sharply and quickly. The CFO-optimized arrays exhibit more moderate FBR values, but over a much greater bandwidth, with the Var Z_0 design performing better than its Fixed Z_0 counterpart.

Input resistance and reactance appear in **Figures 6** and **7**, respectively. The two CFO-optimized arrays exhibit similar behavior for R_{in} with moderate values across the band, whereas the A3's input resistance drops nearly to zero beyond around 310 MHz. For all three antennas, the reactance increases more or less monotonically from about –110 Ω at 200 MHz to between +130 Ω and +130 Ω at 375 MHz. Each Yagi exhibits a single resonance across the entire band.

5. Conclusion

This paper provides an example of applying Central Force Optimization and Variable $Z_0^{(sm)*}$ technology to the design of a wideband Yagi-Uda array. Even though Yagis are generally considered "narrowband" antennas, the CFO-Var Z_0 approach produces a Yagi design with good gain and FBR over a fractional bandwidth greater

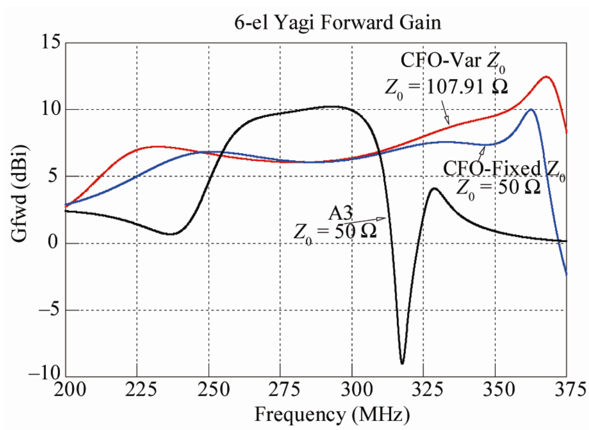


Figure 4. Yagi forward gain.

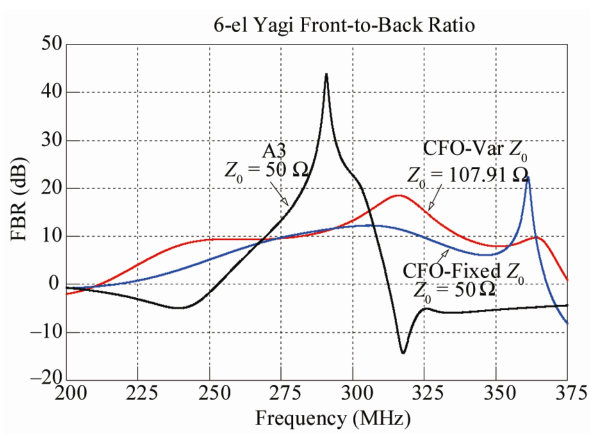


Figure 5. Yagi FBR.

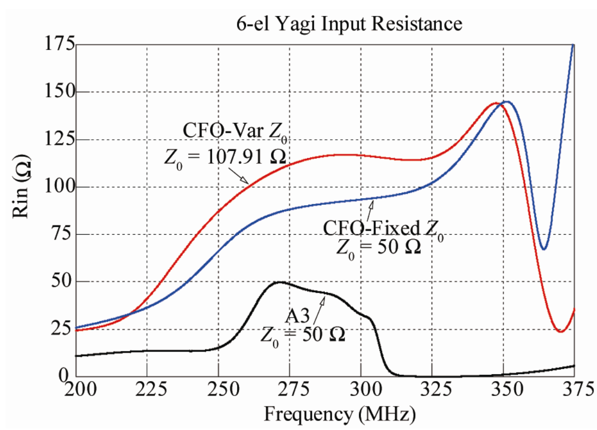


Figure 6. Yagi input resistance.

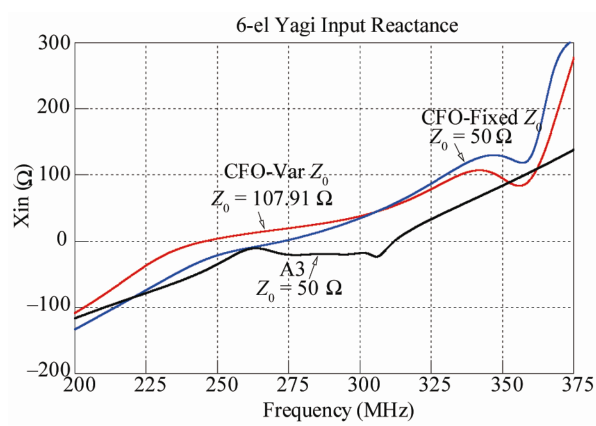


Figure 7. Yagi input reactance.

than 33% with $VSWR \le 2{:}1$. Variable Z_0 is a new approach to antenna design (patent pending) in which the feed system characteristic impedance (or source internal impedance), Z_0, is treated as a variable quantity whose value is determined by the design or optimization methodology. Variable Z_0 is a heretofore overlooked and fundamentally different antenna design methodology that departs from the traditional methodology of treating Z_0 as a fixed design parameter whose value is specified at the outset and never changes. By introducing into the antenna design or decision space an additional degree of freedom, Variable Z_0 makes it easier to achieve specific performance goals, as illustrated by the Yagi design example. While Variable Z_0 should be especially useful for improving IBW, it will be useful in achieving any desired antenna performance objectives, even objectives not involving IBW directly.

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